A BEHAVIOURAL CAR-FOLLOWING MODEL FOR COMPUTER SIMULATION

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Abstract—The ability to predict the response of a vehicle in a stream of traffic to the behaviour of its predecessor is important in estimating what effect changes to the driving environment will have on traffic flow. Various models proposed to explain this behaviour have different strengths and weaknesses. This paper constructs a new model for the response of the following vehicle based on the assumption that each driver sets limits to his desired braking and acceleration rates. The parameters in the model correspond directly to obvious characteristics of driver behaviour and the paper goes on to show that when realistic values are assigned to the parameters in a simulation, the model reproduces the characteristics of real traffic flow.

INTRODUCTION

The way in which one car in a stream of traffic reacts to the behaviour of the preceding vehicle has been the subject of numerous modelling efforts in the past (Gazis *et al.*, 1961; Newell, 1961; Lee, 1966; Bender and Fenton, 1972; to name only a few). The drive to develop a good car-following model arises from the need to analyse the effects on traffic flow of proposed changes to the road network. Most models in current use are variations of

$$a_n(t+\tau) = l_n \frac{[v_{n-1}(t) - v_n(t)]^k}{[x_{n-1}(t) - x_n(t)]^m} \quad (1)$$

where vehicle $n-1$ is followed immediately by vehicle n, and τ is the reaction time, $x_n(t)$ is the location of vehicle n at time t, $v_n(t)$ is the speed of vehicle n at time t, $a_n(t+\tau)$ is the acceleration of vehicle n at time $t+\tau$, and l_n, k and m are parameters that need to be estimated.

While these models have served well in many situations, it is desirable for the interval between successive recalculations of acceleration, speed and location to be a fraction of the reaction time, as pointed out be Seddon (1972). This necessitates the storage of a considerable quantity of historical data if the model is to be used in a simulation program. Further, the parameters l_n, k and m have no obvious connection with identifiable characteristics of driver or vehicle.

This paper therefore looks at the construction of a new car-following model designed to possess the following properties:

(a) the model should mimic the behaviour of real traffic,
(b) the parameters in the model should correspond to obvious characteristics of drivers and vehicles so that most can be assigned values without resorting to elaborate calibration procedures,
and
(c) the model should be well behaved when the interval between successive recalculations of speed and position is the same as the reaction time.

While properties (a) and (c) can only be verified after the model has been used in simulations, (b) can be obtained by constructing the model on an explanatory rather than descriptive basis.

THE MODEL

The model which follows is derived by setting limits on the performance of driver and vehicle and using these limits to calculate a safe speed with respect to the preceding vehicle. It is assumed that the driver of the following vehicle selects his speed to ensure that he can bring his vehicle to a safe stop should the vehicle ahead came to a sudden stop.

In developing the model, the following notation will be used:

- a_n is the maximum acceleration which the driver of vehicle n wishes to undertake,
- b_n is the most severe braking that the driver of vehicle n wishes to undertake ($b_n < 0$),
- s_n is the effective size of vehicle n, that is, the physical length plus a margin into which the following vehicle is not willing to intrude, even when at rest,
- V_n is the speed at which the driver of vehicle n wishes to travel,
- $x_n(t)$ is the location of the front of vehicle n at time t,
- $v_n(t)$ is the speed of vehicle n at time t, and
- τ is the apparent reaction time, a constant for all vehicles.

The first two constraints to be applied to vehicle n are that it will not exceed its driver's desired speed and its free acceleration should first increase with speed as engine torque increases then decrease to zero as the vehicle approaches the desired speed. These two limitations are combined in the inequality

$$v_n(t+\tau) \leq v_n(t) + 2.5a_n\tau(1 - v_n(t)/V_n)(0.025 + v_n(t)/V_n)^{1/2}. \tag{2}$$

This inequality was the result of fitting an envelope to a plot of instantaneous speeds and accelerations obtained from an instrumented car travelling down an arterial road in moderate traffic. The expression is purely descriptive, the precise form being chosen from several equally satisfactory descriptions on the basis that it yielded the simplest calculations. The use of a descriptive expression for this part of the model was considered to be acceptable since it did not impinge on the car following behaviour, only becoming dominant when the preceding vehicle was too distant or travelling too fast to have any effect.

The next limitation to be considered is braking. If vehicle $n-1$ commences braking as hard as desirable at time t, it will come to rest at point x^*_{n-1} given by

$$x^*_{n-1} = x_{n-1}(t) - v_{n-1}(t)^2/2b_{n-1} \tag{3}$$

since b_{n-1} is negative.

Vehicle n travelling immediately behind will not react until time $t+\tau$ and consequently will not come to rest before reaching x^*_n, given by

$$x^*_n = x_n(t) + [v_n(t) + v_n(t+\tau)]\tau/2 - v_n(t+\tau)^2/2b_n. \tag{4}$$

Thus for safety, the driver of vehicle n must ensure that $x^*_{n-1} - s_{n-1}$ exceeds x^*_n. However, if this were the governing inequality, the driver of vehicle n would have no margin for error. Therefore, let us introduce a further safety margin by supposing that the driver makes allowance for a possible additional delay of θ when he will be travelling at $v_n(t+\tau)$, before reacting to the vehicle ahead. That is, there is a true reaction time, τ, and a safety reaction time, $\tau + \theta$, to appear in the calculations. Thus the limitation on braking requires that

$$x_{n-1}(t) - v_{n-1}(t)^2/2b_{n-1} - s_{n-1} \geq x_n(t) + [v_n(t) + v_n(t+\tau)]\tau/2 + v_n(t+\tau)\theta - v_n(t+\tau)^2/2b_n. \tag{5}$$

Without the introduction of the parameter θ in this fashion, a single vehicle approaching a stationary object or a stop line would travel at its desired speed until it had to commence maximum braking. The effect of θ is to cause the simulated vehicle to brake earlier and to gradually reduce braking so that it crawls up to the stop line.

In real traffic, it is possible for the driver of vehicle n to estimate all the values in (5) except b_{n-1} by direct observation. Thus b_{n-1} should be replaced by some estimate $\hat{b}$ to give

$$-v_n(t+\tau)^2/2b_n + v_n(t+\tau)(\tau/2+\theta) - [x_{n-1}(t) - s_{n-1} - x_n(t)] + v_n(t)\tau/2 + v_{n-1}(t)^2/2\hat{b} \leq 0. \tag{6}$$

The relative magnitudes of τ and θ are important in determining the behaviour of vehicles. It can be shown (Appendix) that, if θ is equal to $\tau/2$ and the willingness of the previous driver to brake hard has not been underestimated, a vehicle travelling at a safe speed and distance will be able to maintain a state of safety indefinitely. Thus (6) can be rewritten as

$$-v_n(t+\tau)^2/2b_n + v_n(t+\tau)\tau - [x_{n-1}(t) - s_{n-1} - x_n(t)] + v_n(t)\tau/2 + v_{n-1}(t)^2/2\hat{b} \leq 0. \quad (7)$$

Hence

$$v_n(t+\tau) \leq b_n\tau + \sqrt{(b_n^2\tau^2 - b_n(2[x_{n-1}(t) - s_{n-1} - x_n(t)] - v_n(t)\tau - v_{n-1}(t)^2/\hat{b}))}. \quad (8)$$

The inequality in (7) implies that safe speeds lie between the two roots of the equation, but since the lower root is negative, it can be disregarded as we are only interested in positive speeds. The possibility that the driver of vehicle n has underestimated the readiness of the preceding driver to brake hard with the result that under severe braking $v_n(t+\tau)$ is less than $v_n(t) + b_n\tau$, can be handled if it is assumed that vehicle n is capable of more severe braking than the driver wishes to undertake. That is, the driver selects his speed with respect to a desired (most severe) braking rate, but can brake harder if necessary.

Equations (2) and (8) represent two constraints on the speed of vehicle n at time $t+\tau$ and, if it is assumed that the driver travels as fast as safety and the limitations of the vehicle permit, the new speed is given by

$$v_n(t+\tau) = \min\{v_n(t) + 2.5a_n\tau(1 - v_n(t)/V_n)(0.025 + v_n(t)/V_n)^{1/2},$$

$$b_n\tau + \sqrt{(b_n^2\tau^2 - b_n[2[x_{n-1}(t) - s_{n-1} - x_n(t)] - v_n(t)\tau - v_{n-1}(t)^2/\hat{b}])}\}. \quad (9)$$

When (8) is the limiting condition for almost all vehicles, congested flow exists with the traffic flowing as fast as the volume of vehicles permit. When (2) is the limiting condition for a substantial proportion of the vehicles, the traffic flows freely.

The transition of $v_n(t+\tau)$ between the two terms in (9) occurs smoothly as the limitation imposed by the second (safety) term has an effect some time before maximum braking would be required. It is possible for a vehicle to continue to accelerate even after a transition from the acceleration limitation to the braking limitation. The only applications of the model in which a smooth transition does not necessarily occur are when the leading vehicle brakes harder than the following vehicle has anticipated or leaves the lane, or when a new vehicle moves into the gap between the two vehicles. However, these are the circumstances under which real traffic may also exhibit a rapid transition between acceleration and braking.

VALIDATION OF THE MODEL

The model as constructed satisfies the criterion that the parameters should correspond to obvious characteristics of drivers and vehicles. However, it remains to be shown that when reasonable values are assigned to these parameters, the model is able to mimic the behaviour of real traffic.

As a test of the capabilities of the model, a trial simulation was run with the parameters for vehicle n selected by:

- a_n sampled from a normal distribution, $N(1.7, 0.3^2)$ m/sec^2,
- b_n equated to $-2.0a_n$,
- s_n sampled from a normal population $N(6.5, 0.3^2)$ m,
- V_n sampled from a normal population, $N(20.0, 3.2^2)$ m/sec,
- τ 2/3 second, and
- $\hat{b}$ minimum of -3.0 and $(b_n - 3.0)/2$ m/sec^2.

An improved version of the program MULTSIM (Gipps, 1976) was used to simulate three lanes of a divided highway. Vehicles passing a fixed point during one-minute intervals were timed over the next 100 metres to yield volume and speed measurements. The results of these measurements are displayed in Fig 1 and show the typical "horse-shoe" shape observed in practice. Experimentation with these parameters suggested that the flow-speed curve was relatively insensitive to a_n, b_n and s_n, but could be adjusted by changes to V_n, τ and $\hat{b}$. The

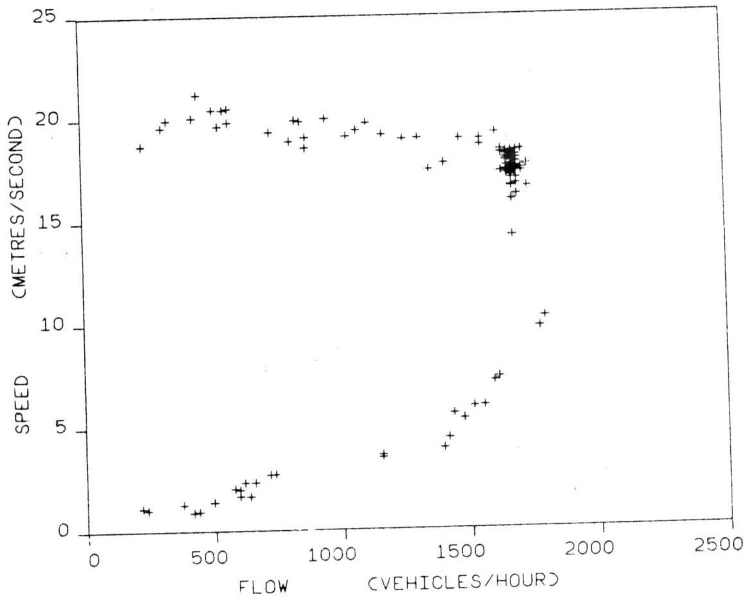

Fig. 1. Speed-flow observations obtained from a simulation program using the car-following model.

mean and standard deviation of the distribution of desired speeds affected the position and shape of the upper arm of the speed-flow curve, while τ and $\hat{b}$ were responsible for determining the maximum flow.

Another effect of $\hat{b}$, not obvious in the flow-speed curve, was that it determined whether disturbances to the traffic flow were damped or amplified as they were transmitted to successive vehicles. If $\hat{b}$ was less than b_{n-1} (that is, willingness of driver $n-1$ to brake was over-estimated), disturbances were damped, but if $\hat{b}$ was greater than b_{n-1}, disturbances could be amplified. This is illustrated in Figs 2 and 3. Both figures show the speed-time curves for seven successive vehicles when the speed of the first vehicle is perturbed. The seven vehicles are identical and the parameters are:

$$a_n = 2.0 \text{ m/sec}^2,$$

$$b_n = -3.0 \text{ m/sec}^2,$$

$$V_n = 20.0 \text{ m/sec, and}$$

$$\tau = 2/3 \text{ sec.}$$

In Fig. 2, $\hat{b}$ was set equal to -3.5 m/sec^2, and the disturbance was damped, but when $\hat{b}$ was set equal to -2.5 m/sec^2 (Fig. 3), the disturbance was propagated.

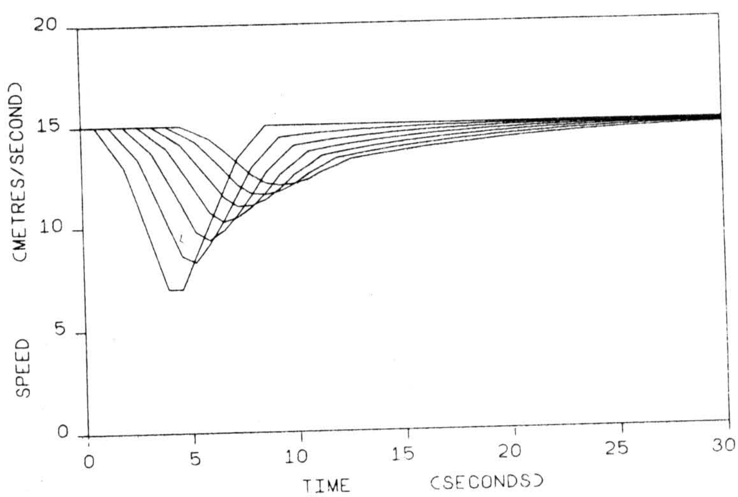

Fig. 2. Speed-time plots for seven successive vehicles when disturbances to the traffic flow are damped.

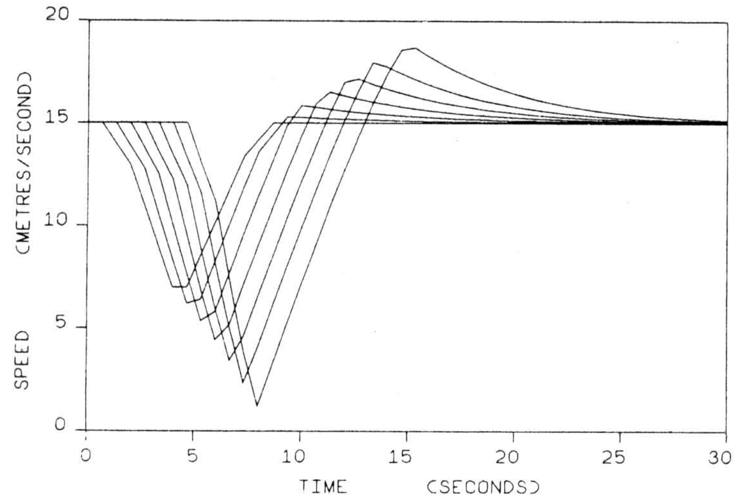

Fig. 3. Speed-time plots for seven successive vehicles when disturbances to the flow are amplified.

CONCLUSIONS

The model of car-following behaviour proposed appears to be able to mimic the behaviour of real traffic, and the parameters involved correspond to obvious driver and vehicle characteristics and affect the behaviour of the simulated flow in logically consistent ways. The relative insensitivity of the speed-flow curve to a_n is not surprising. Only the first one or two vehicles at the head of a platoon are in a position to accelerate as hard as they like; the others are constrained by the presence of the vehicle ahead. A similar situation exists with respect to braking, where the later vehicles in a platoon do not reach their braking limits unless they have seriously underestimated the willingness of their predecessors to brake. Errors in estimating s_n, the effective length of vehicles, are not likely to cause major errors in speed or flow characteristics because s_n is very much smaller than the distance separating vehicles, except when traffic is nearly stationary.

Thus the corporate behaviour of the traffic is principally controlled by three factors: (a) the distribution of desired speeds, V_n, (b) the reaction time for drivers, τ, and (c) the ratio of mean braking rate to driver's estimates of the mean braking rate, $\bar{b}/\hat{b}$, while the distributions of acceleration, a_n, braking, b_n, and effective length, s_n, govern the individual behaviour of vehicles.

Experience in applying the model in simulation shows it to be well behaved when the interval between successive recalculations of speed and position is the same as the reaction time, τ, thus fulfilling the third of the criteria set in the Introduction.

An additional advantage of the model is its speed of calculation. Because (8) contains square roots and squares but not general powers of variables, it is relatively fast to evaluate. A comparison with (1) using general exponents for k and m showed the car following model derived in this paper to be about 15 per cent faster per evaluation. This comparison is even more favourable when it is recalled that (1) needed to be evaluated several times per reaction time, not just once.

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APPENDIX

In the derivation of the car-following model it was asserted that provided θ was equal to $\tau/2$ and the willingness of the preceding driver to brake hard had not been underestimated, a vehicle travelling at a safe speed would be able to maintain a safe speed and distance indefinitely. The formal proof of this assertion is as follows:

Assertion

If

$$-v_n(t+\tau)^2/2b_n + v_n(t+\tau)(\tau/2+\theta) - [x_{n-1}(t) - s_{n-1} - x_n(t)] + v_n(t)\tau/2 + v_{n-1}(t)^2/2\hat{b} \leqslant 0, \quad \text{(A1)}$$

and

$$\theta = \tau/2,$$

and

$$v_{n-1}(t+\tau) \geqslant v_n(t+\tau) + \hat{b}\tau, \quad \text{(A2)}$$

then there always exists some $v_n(t+2\tau)[\geqslant v_n(t+\tau)+b_n\tau]$ which satisfies

$$-v_n(t+2\tau)^2/2b_n + v_n(t+2\tau)(\tau/2+\theta) - [x_{n-1}(t+\tau) - s_{n-1} - x_n(t+\tau)] + v_n(t+\tau)\tau/2 + v_{n-1}(t+\tau)^2/2\hat{b} \leqslant 0. \quad \text{(A3)}$$

Proof

Let us equate the left hand side of (A3) to F and subtract (A1)

$$\begin{aligned} F \leqslant & -v_n(t+2\tau)^2/2b_n + v_n(t+2\tau)(\tau/2+\theta) - [x_{n-1}(t+\tau) - s_{n-1} - x_n(t+\tau)] \\ & + v_n(t+\tau)/2 + v_{n-1}(t+\tau)^2/2\hat{b} \\ & + v_n(t+\tau)^2/2b_n - v_n(t+\tau)(\tau/2+\theta) + [x_{n-1}(t) - s_{n-1} - x_n(t)] \\ & - v_n(t)\tau/2 - v_{n-1}(t)^2/2\hat{b}. \end{aligned}$$

Now from (A2) and the assumption of uniform deceleration from t to $t+\tau$

$$x_{n-1}(t+\tau) - v_{n-1}(t+\tau)^2/2\hat{b} \geqslant x_{n-1}(t) - v_{n-1}(t)^2/2\hat{b}.$$

Therefore,

$$\begin{aligned} F \leqslant & -v_n(t+2\tau)^2/2b_n + v_n(t+2\tau)(\tau/2+\theta) + v_n(t+\tau)\tau/2 + v_n(t+\tau)^2/2b_n \\ & - v_n(t+\tau)(\tau/2+\theta) - v_n(t)\tau/2 + [x_n(t+\tau) - x_n(t)]. \end{aligned}$$

Now,

$$x_n(t+\tau) - x_n(t) = [v_n(t+\tau) + v_n(t)]\tau/2.$$

Hence,

$$F \leqslant [v_n(t+\tau)^2 - v_n(t+2\tau)^2]/2b_n + v_n(t+2\tau)(\tau/2+\theta) + v_n(t+\tau)(\tau/2-\theta).$$

There are now two possible cases to consider, depending on whether $v_n(t+2\tau)$ is greater than or equal to zero.

Case 1

$$v_n(t+2\tau) = v_n(t+\tau) + b_n\tau \geqslant 0,$$

$$F \leqslant -b_n\tau(2v_n(t+\tau) + b_n\tau)/2b_n + (v_n(t+\tau) + b_n\tau)(\tau/2+\theta) + v_n(t+\tau)(\tau/2-\theta),$$

that is,

$$F \leqslant b_n \tau \theta.$$

Thus, F will always be less than or equal to zero provided θ is positive.

Case 2

$$v_n(t+2\tau) = 0,$$

$$F \leqslant v_n(t+\tau)^2/2b_n + v_n(t+\tau)(\tau/2 - \theta). \tag{A4}$$

Now the right hand side of (A4) reaches a maximum when $v_n(t+\tau)$ equals $-b_n(\tau/2-\theta)$. Thus

$$F \leqslant -b_n(\tau/2-\theta)^2/2,$$

from which it can be seen that F will always be less than or equal to zero provided θ equals $\tau/2$.