Aprendizaje profundo

Perceptrón multicapa

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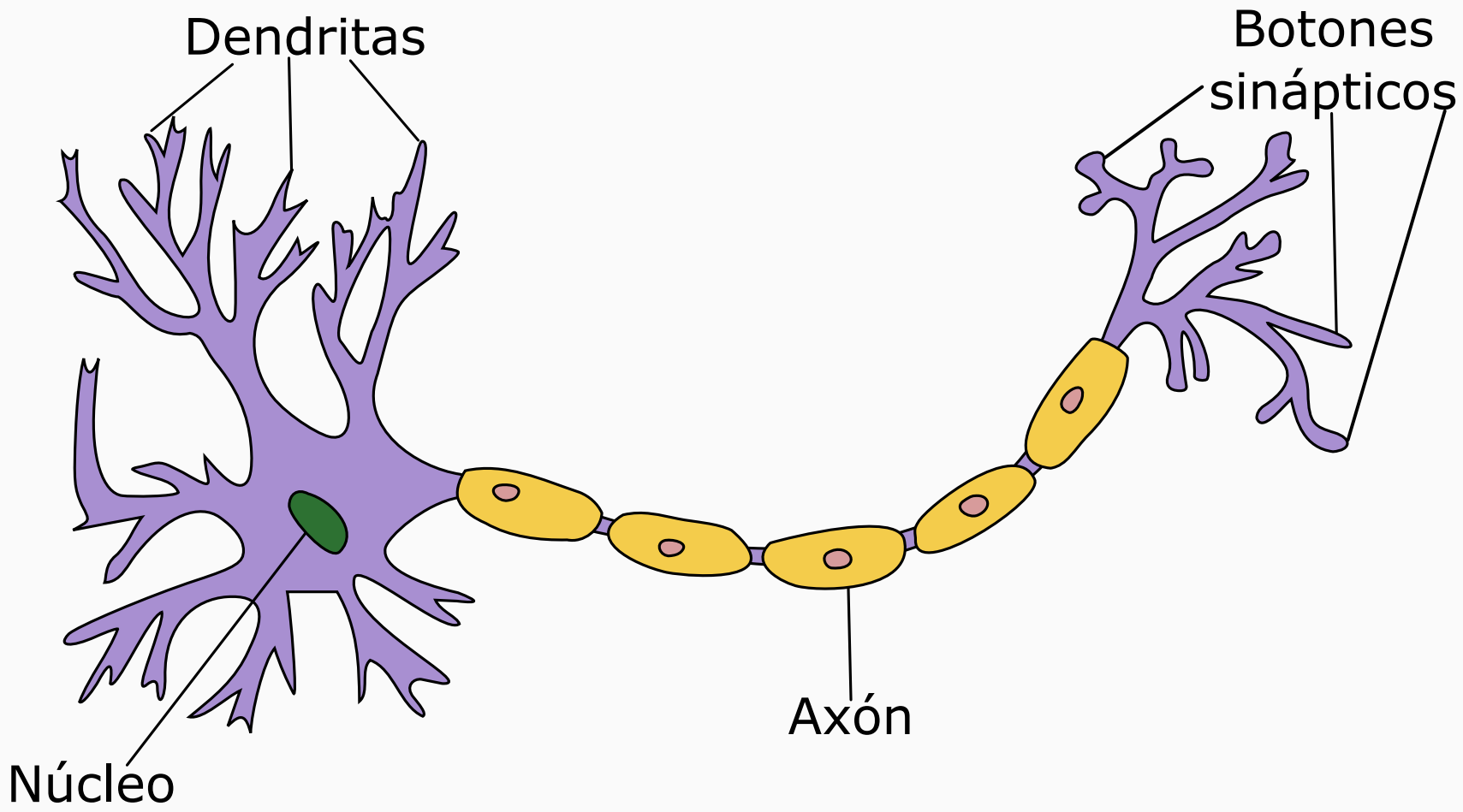
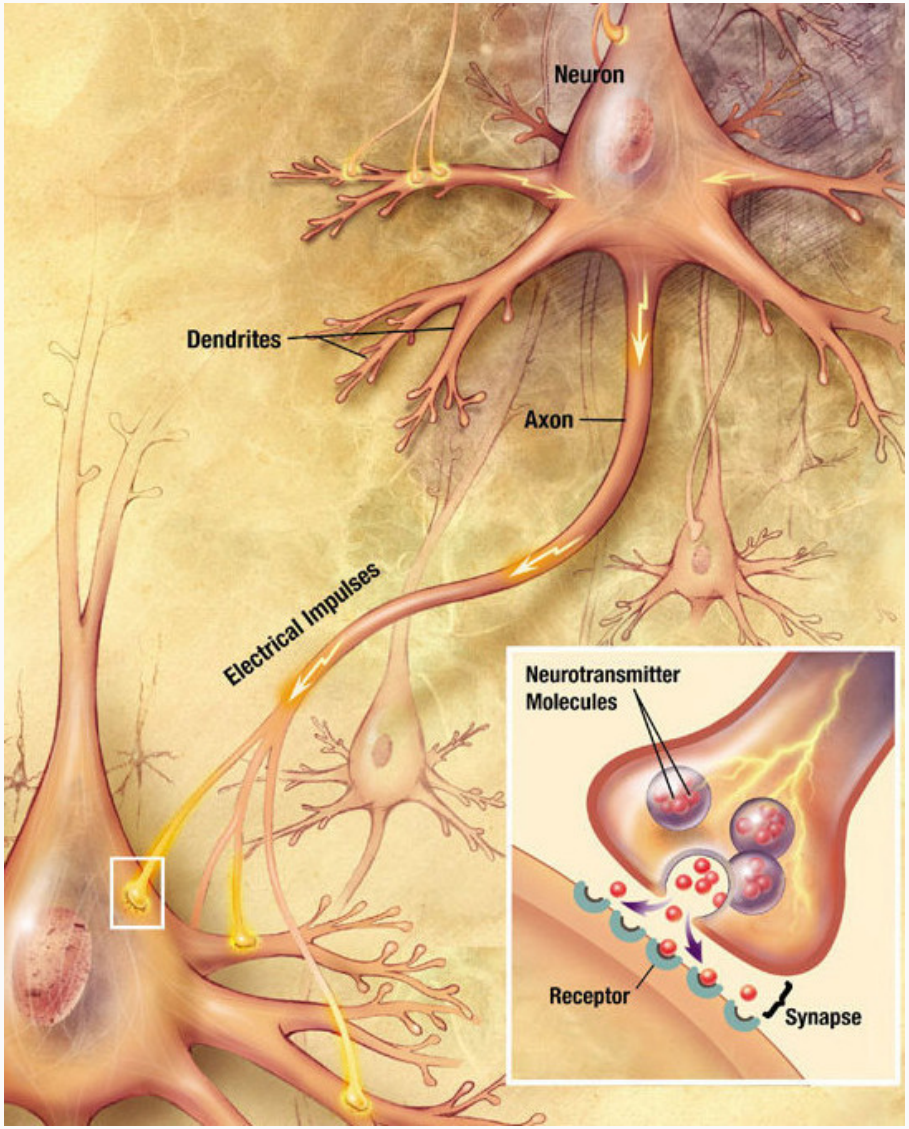
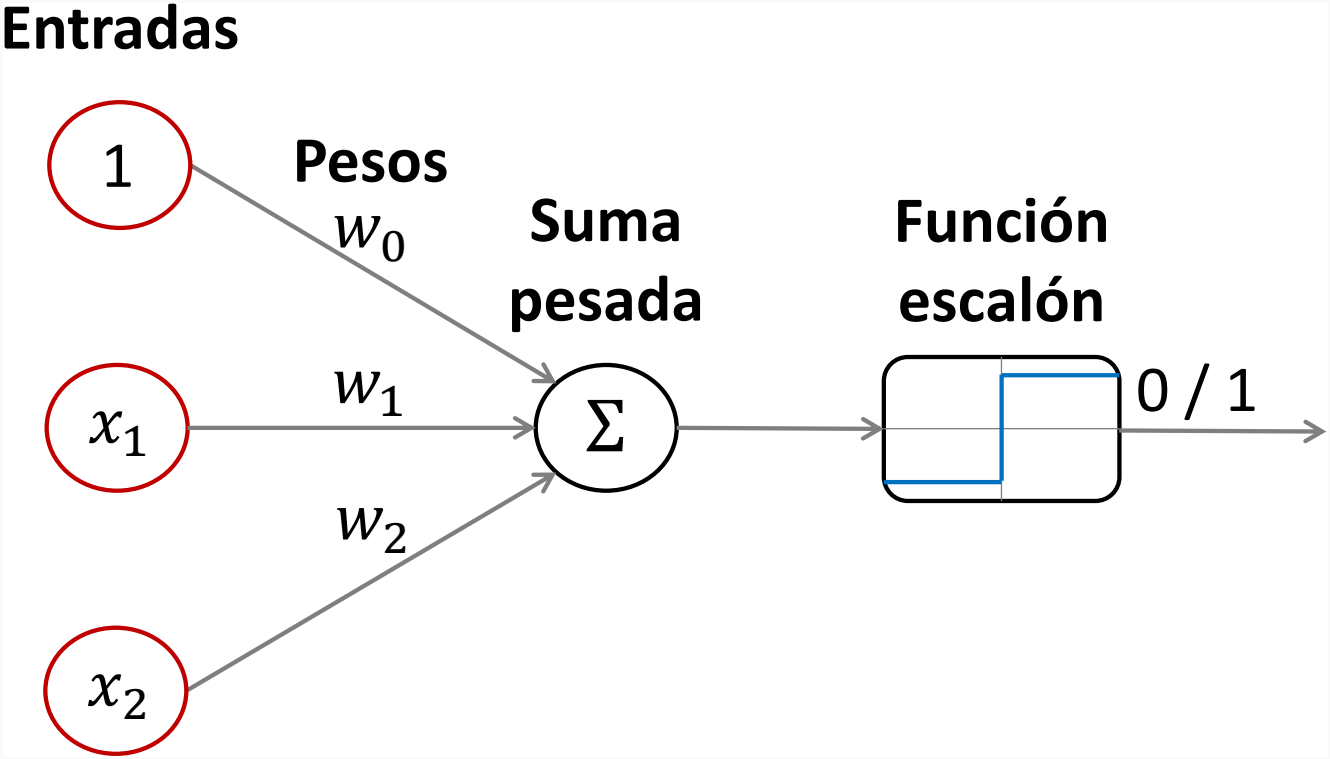
Neurona natural



Comunicación entre neuronas



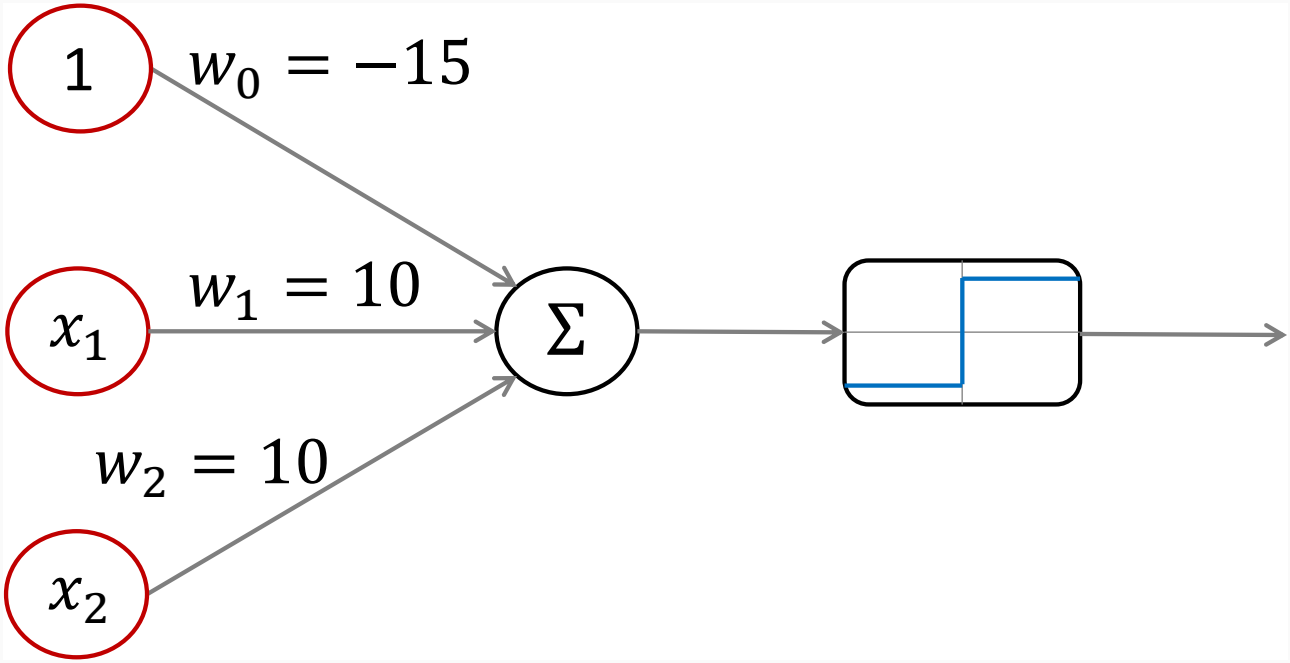
Neurona artificial



- Elementos básicos
  1. Pesos sinápticos
  2. Estímulo cumulativo
  3. Todo o nada (activación)

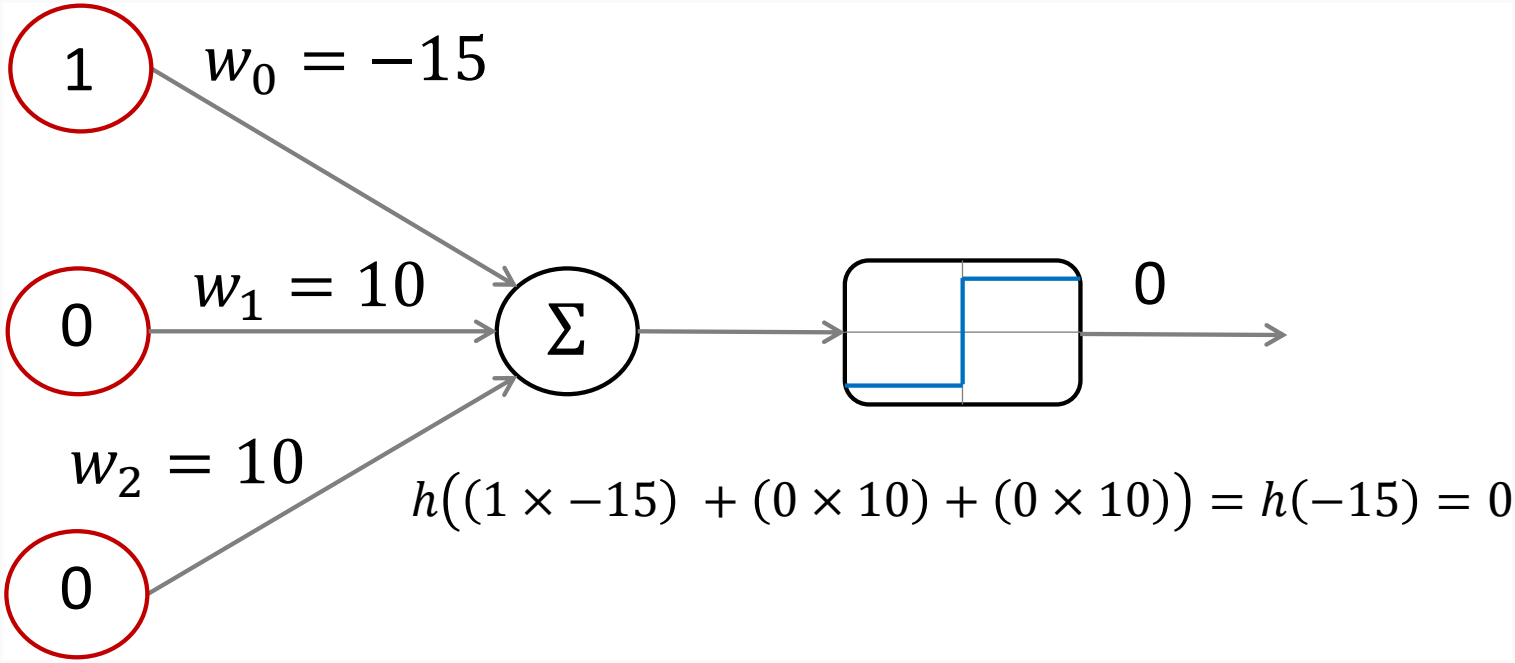
$$\hat{y} = a = \phi(b + \sum_{i=1}^{m} w_i \cdot x_i)$$
$$= \phi(b + \mathbf{w}^\top \cdot \mathbf{x})$$

Compuerta AND ($\wedge$)



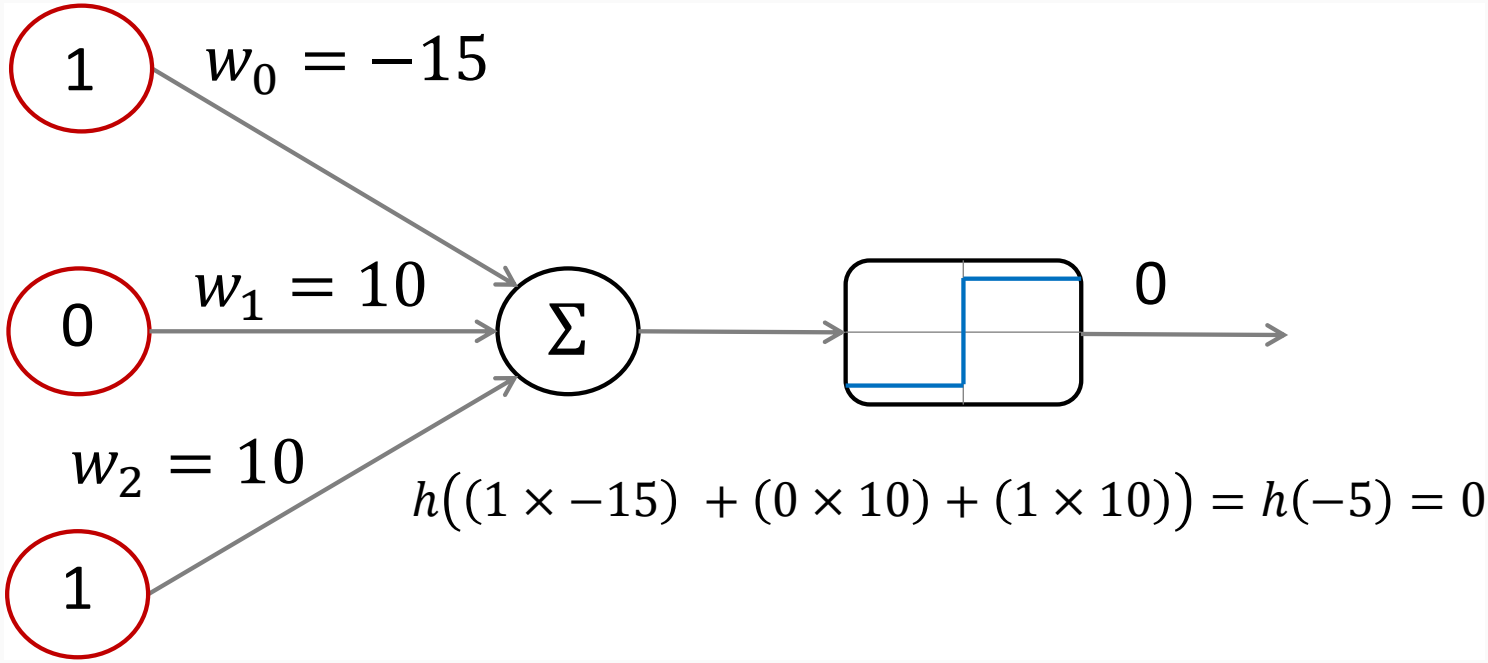
x_1	x_2	AND ($\wedge$)
0	0	0
0	1	0
1	0	0
1	1	1

Compuerta AND ($\wedge$)



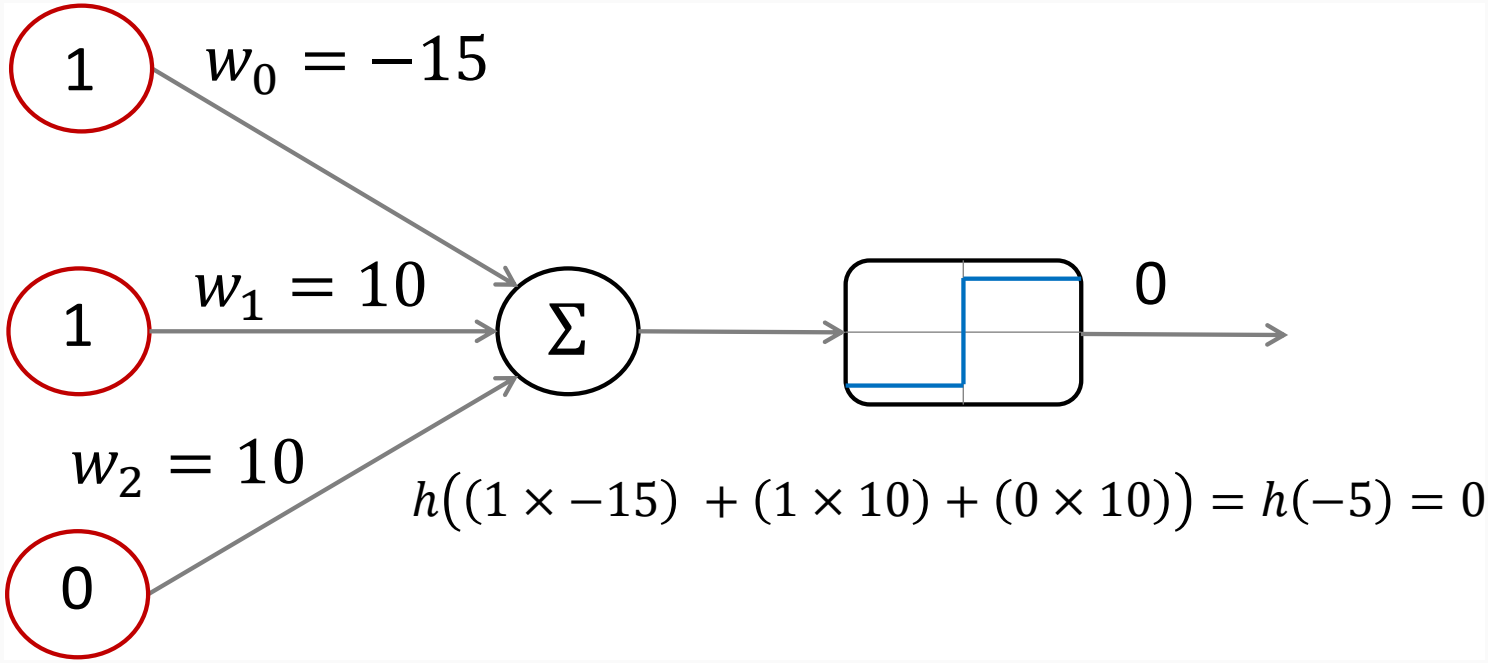
x_1	x_2	AND ($\wedge$)
0	**0**	**0**
0	1	0
1	0	0
1	1	1

Compuerta AND ($\wedge$)



x_1	x_2	AND ($\wedge$)
0	0	0
0	**1**	**0**
1	0	0
1	1	1

Compuerta AND ($\wedge$)



x_1	x_2	AND ($\wedge$)
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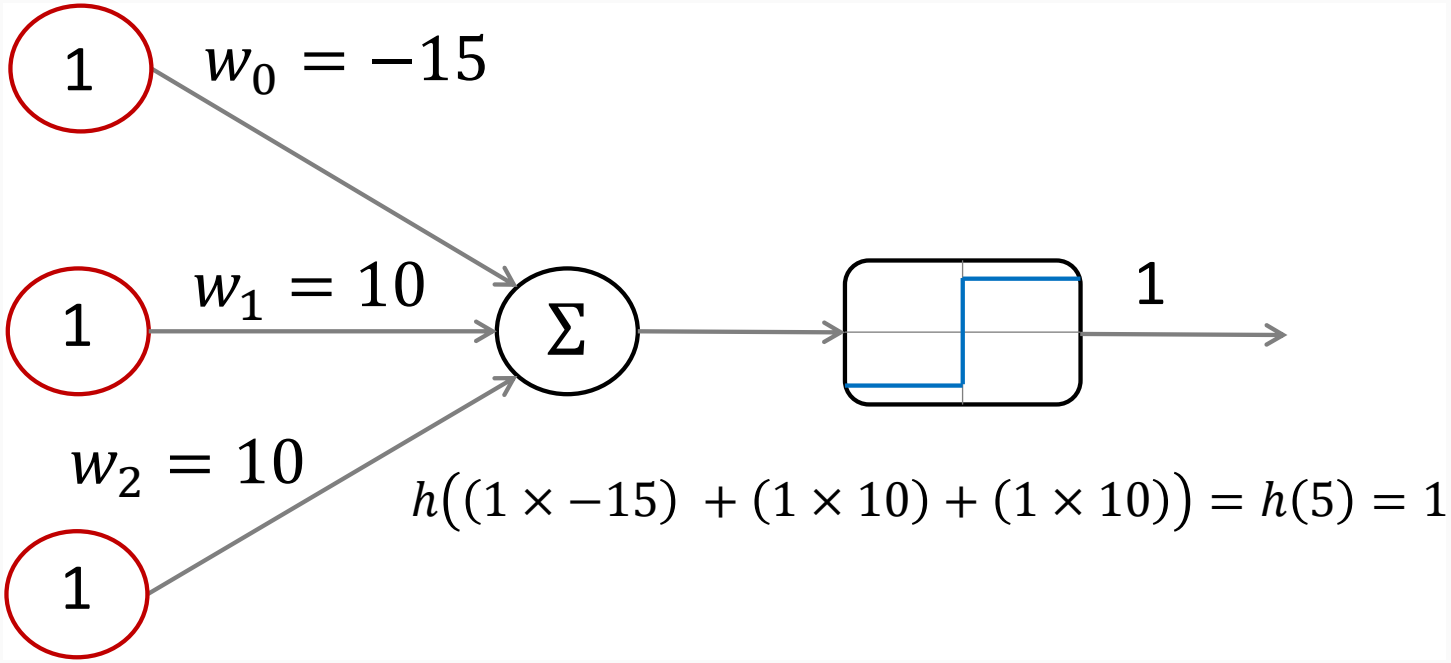
0	1	0
1	**0**	**0**
1	1	1

Compuerta AND ($\wedge$)



x_1	x_2	AND ($\wedge$)
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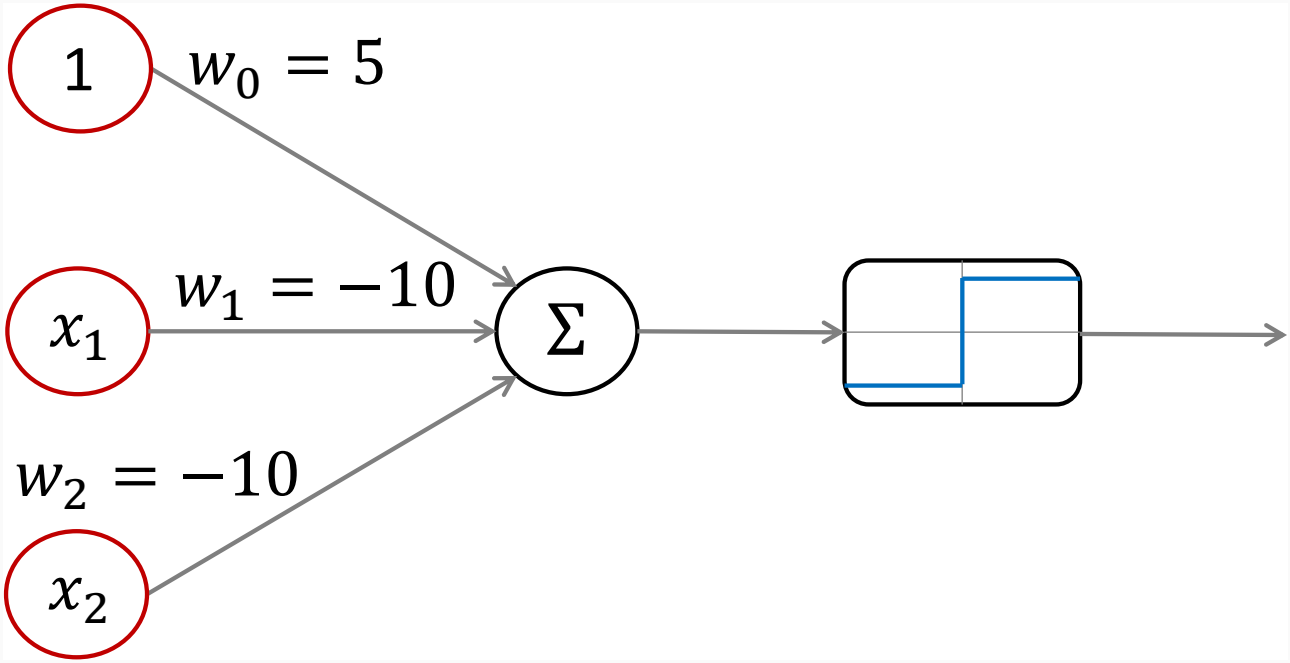
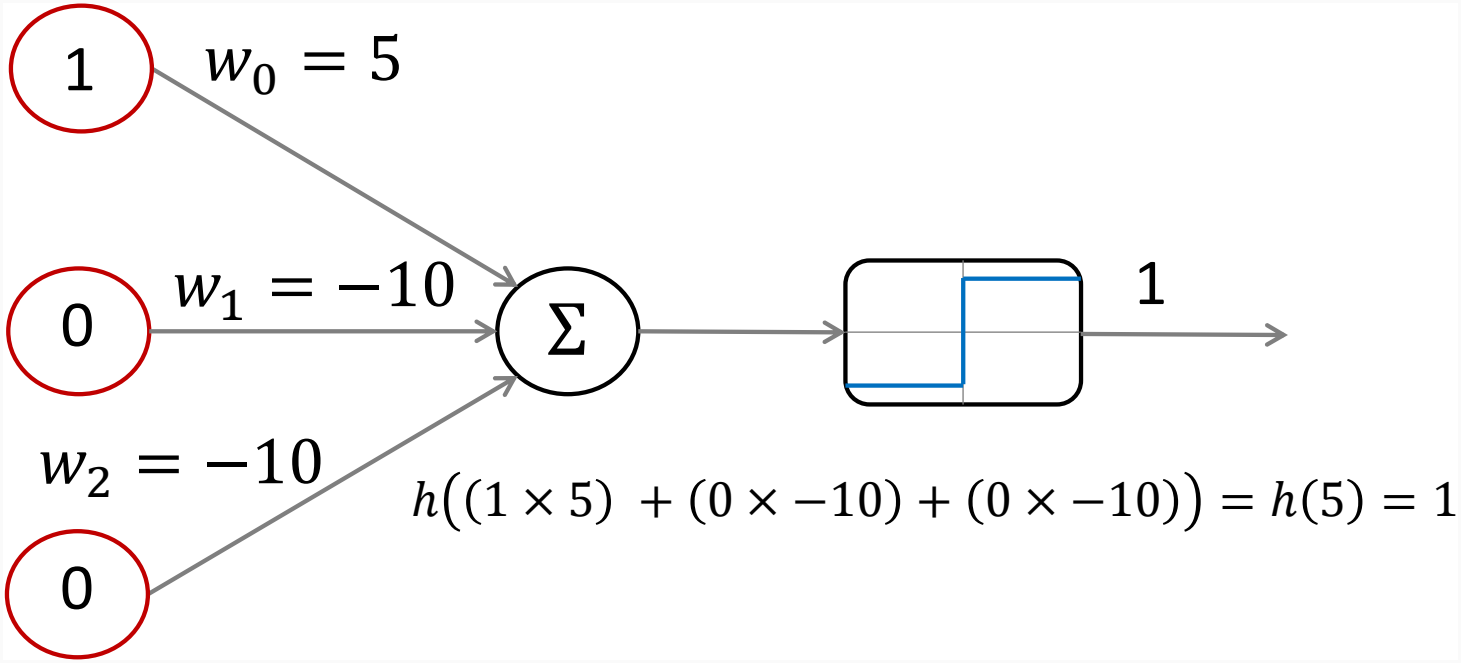
0	1	0
1	0	0
1	**1**	**1**

Compuerta NOR (↓)



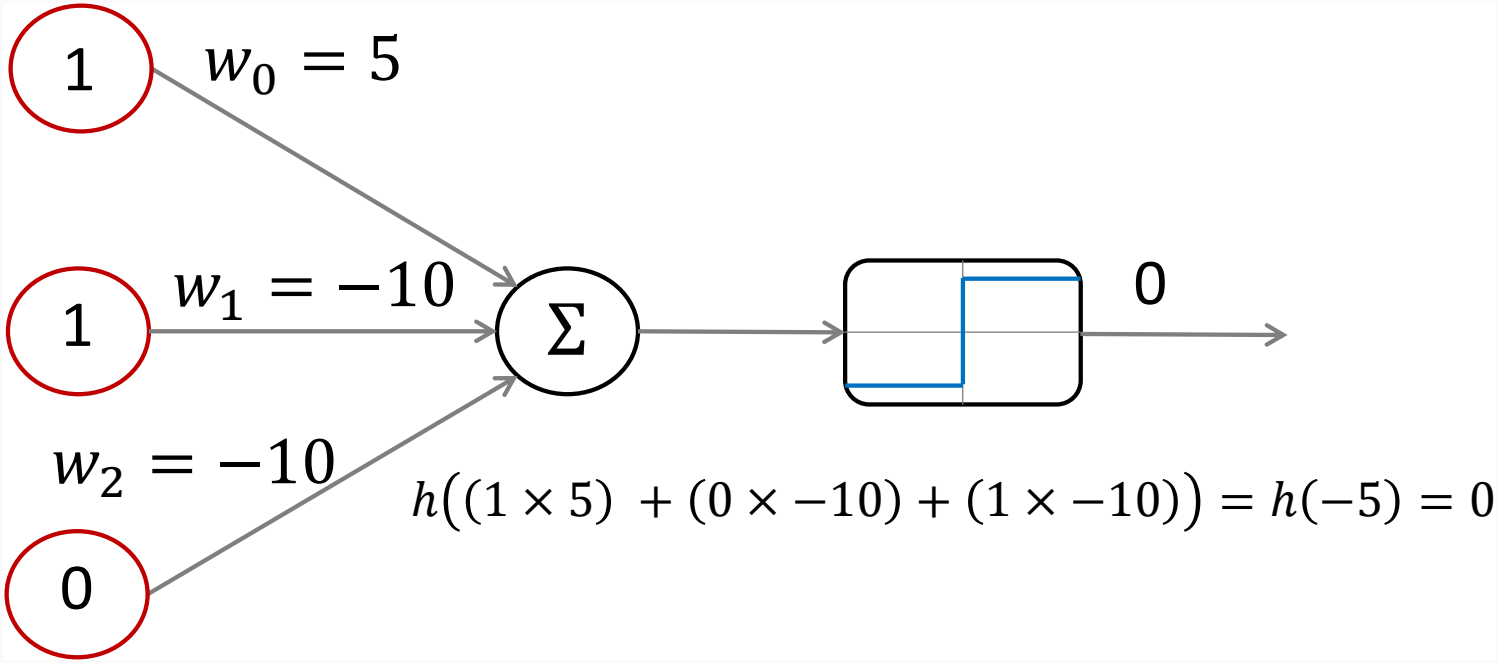
x_1	x_2	NOR (↓)
0	0	1
0	1	0
1	0	0
1	1	0

Compuerta NOR (↓)



x_1	x_2	NOR (↓)
0	**0**	**1**
0	1	0
1	0	0
1	1	0

Compuerta NOR (↓)



x_1	x_2	NOR (↓)
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0	**1**	**0**
1	0	0
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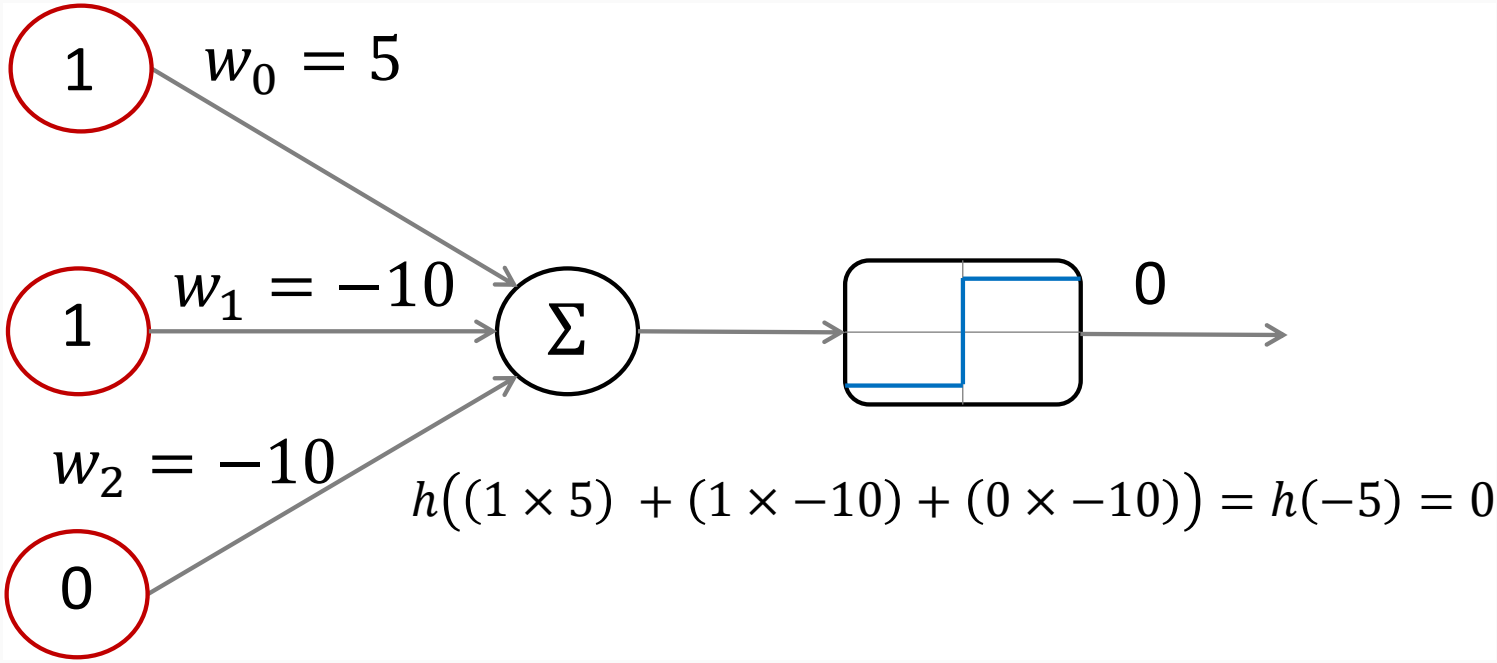
Compuerta NOR (↓)



x_1	x_2	NOR (↓)
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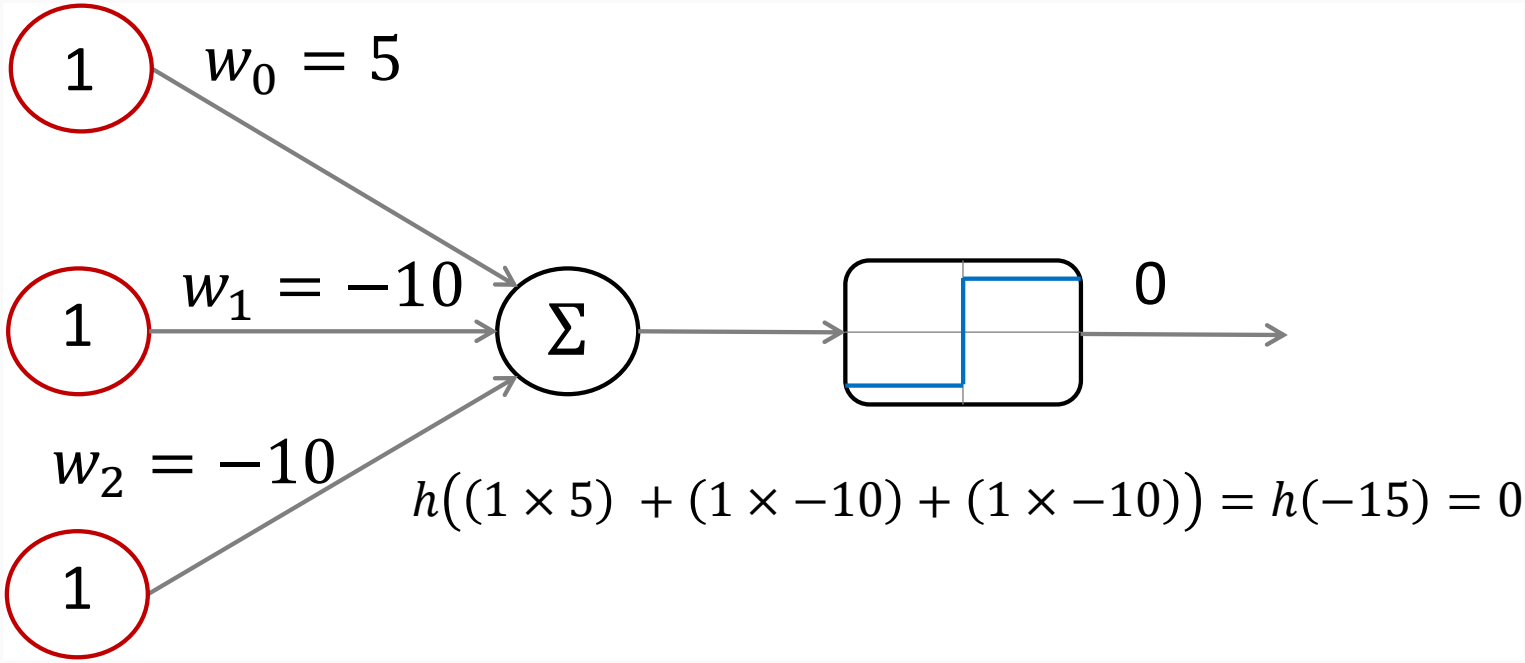
0	1	0
1	**0**	**0**
1	1	0

Compuerta NOR (↓)



x_1	x_2	NOR (↓)
0	0	1
0	1	0
1	0	0
1	**1**	**0**

Algoritmo de aprendizaje: perceptrón

1. Inicializa pesos y sesgo con zeros o un número aleatorio pequeño
2. Para cada ejemplo en el conjunto de entrenamiento
   2.1 Calcula la salida

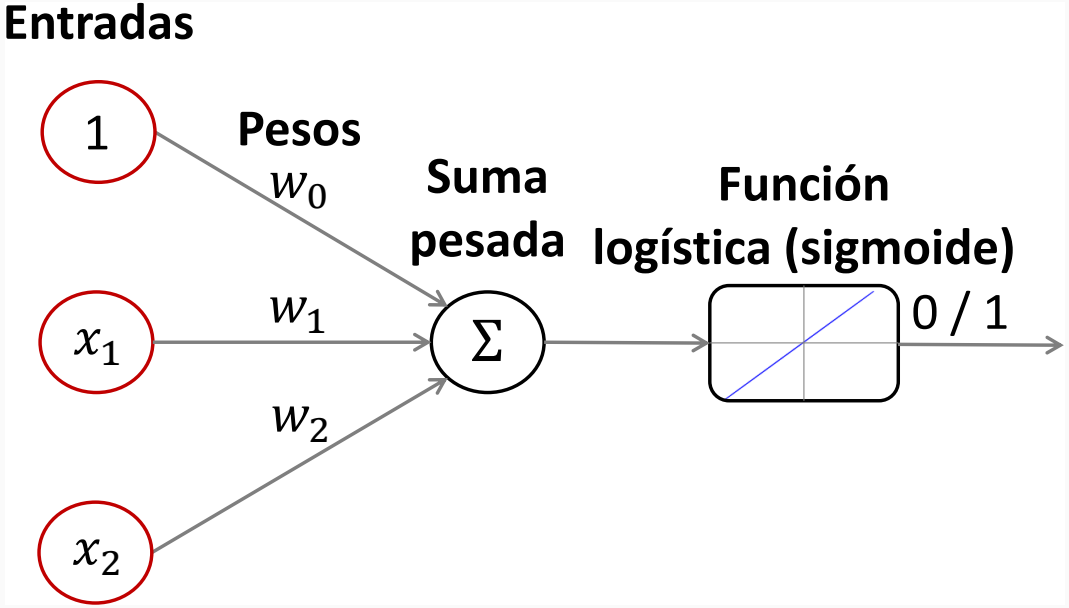
$$\hat{y}^{(i)} = H(\mathbf{w}(t)^\top \mathbf{x}^{(i)} + b)$$

   2.2 Actualiza cada peso $w_j, j = 1, \ldots, d$ y el sesgo b

$$\begin{aligned} w_j[t+1] &= w_j[t] + (y^{(i)} - \hat{y}^{(i)}) \cdot x_j^{(i)} \\ b[t+1] &= b[t] + (y^{(i)} - \hat{y}^{(i)}) \end{aligned}$$

3. Realiza hasta que converja (e.g. error sea menor a umbral)

Neurona con función de activación lineal o identidad



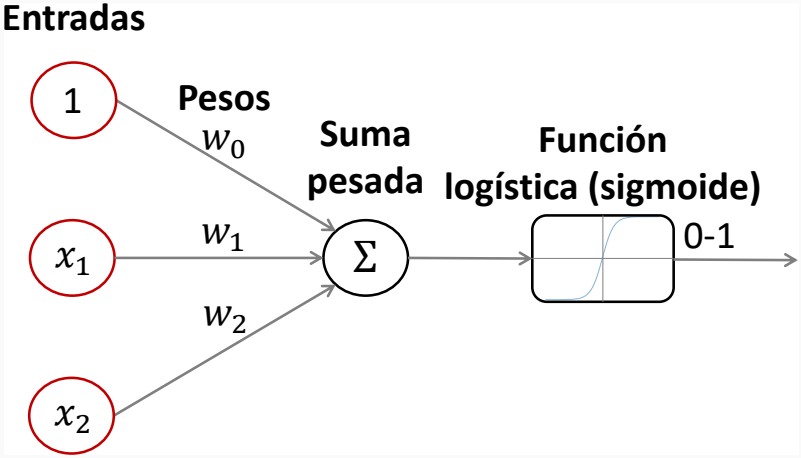
$$lineal(z) = z$$

$$\frac{dlineal(z)}{dz} = 1$$

- Función de pérdida: error cuadráticos medio (ECM)

$$ECM(\mathbf{y}, \hat{\mathbf{y}}) = \frac{1}{2}\sum_{i=1}^{N}(y^{(i)} - \hat{y}^{(i)})^2$$

Neurona con función de activación sigmoide o logística



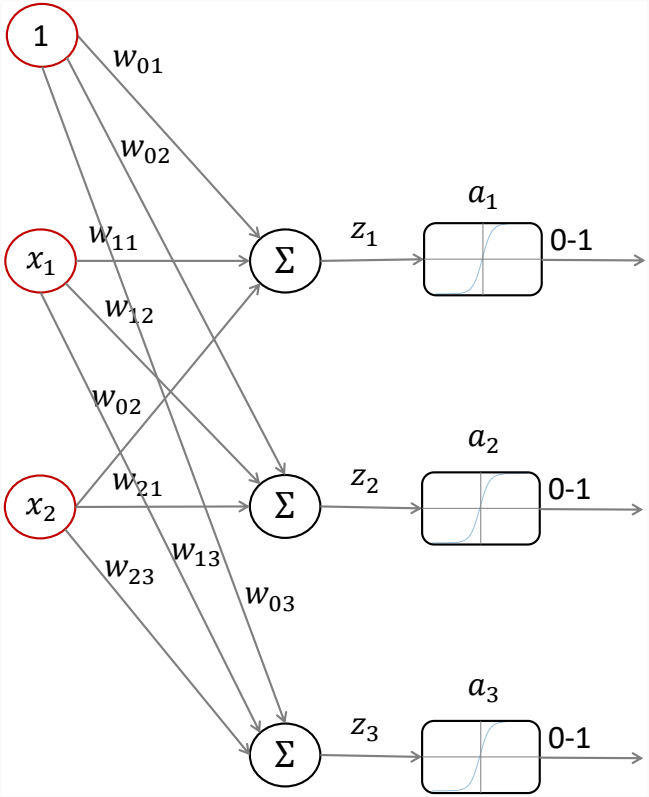
$$sigm(z) = \frac{1}{1 + exp(-z)}$$

$$\frac{dsigm(z)}{dz} = sigm(z)(1 - sigm(z))$$

- Función de pérdida: entropía cruzada binaria (ECB)

$$ECB(\mathbf{y}, \hat{\mathbf{y}}) = -\sum_{i=1}^{N} \left[y^{(i)} \log \hat{y}^{(i)} + (1 - y^{(i)}) \log(1 - \hat{y}^{(i)}) \right]$$

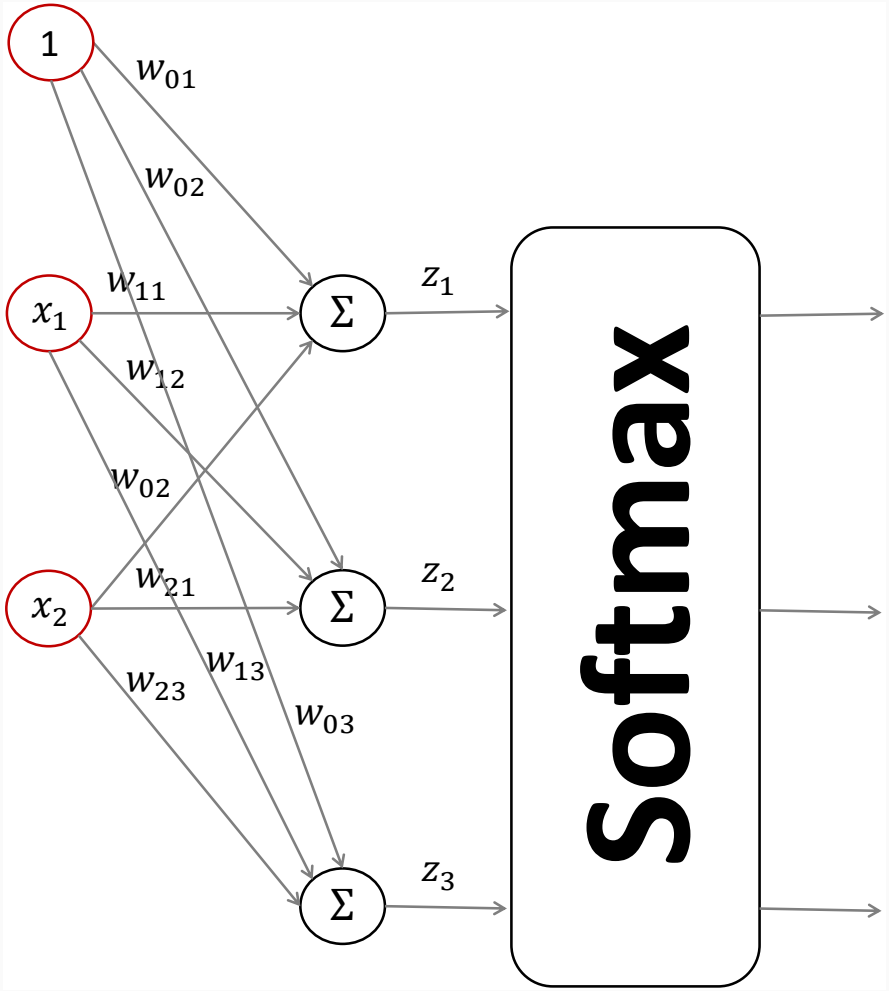
Clasificación multietiqueta



- Función de pérdida: ECB de cada categoría

$$ECB(\mathbf{y}_k, \hat{\mathbf{y}}_k) = -\sum_{i=1}^{N} \left[y_k^{(i)} \log \hat{y}_k^{(i)} + (1 - y_k^{(i)}) \log(1 - \hat{y}_k^{(i)}) \right]$$

Clasificación softmax (1)



Clasificación softmax (2)

- Neuronas de la capa de salida tienen una función de activación *softmax* compartida, dada por

$$softmax(\mathbf{z})_i = \frac{e^{z_i}}{\sum_{j=1}^{K} e^{z_j}}, i = 1, \ldots, K$$

- Función de pérdida de clasificación multiclase: entropía cruzada categórica

$$E(\mathbf{Y}, \hat{\mathbf{Y}}) = -\sum_{i=1}^{N}\sum_{k=1}^{K}\left[1\{y^{(i)} = k\} \log \frac{e^{z_k}}{\sum_j e^{z_j}}\right]$$

Entrenando el modelo: algoritmo del gradiente descendente

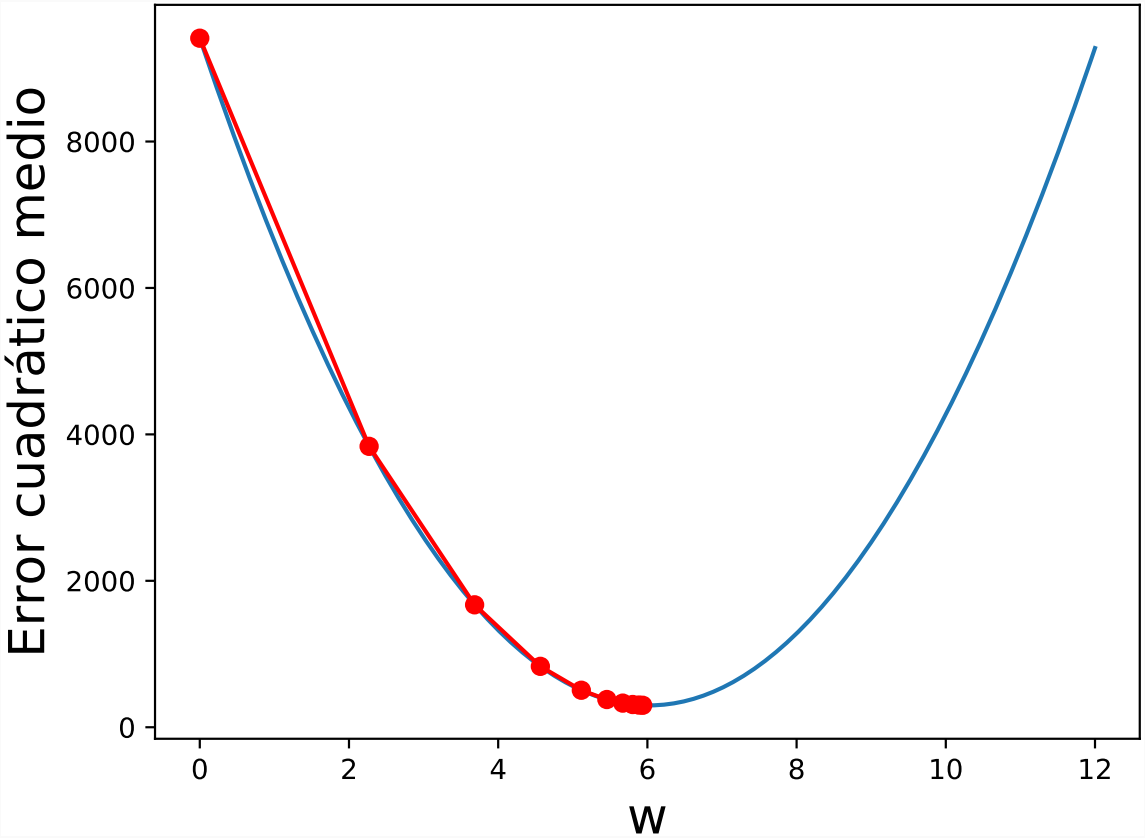
- Mueve sucesivamente pesos y sesgo ($\boldsymbol{\theta} = \{\mathbf{w}, \mathbf{b}\}$) hacia donde más descienda la pérdida

$$\boldsymbol{\theta}^{(k+1)} = \boldsymbol{\theta}^{(k)} - \alpha \mathbf{g}(\boldsymbol{\theta}^{(k)})$$

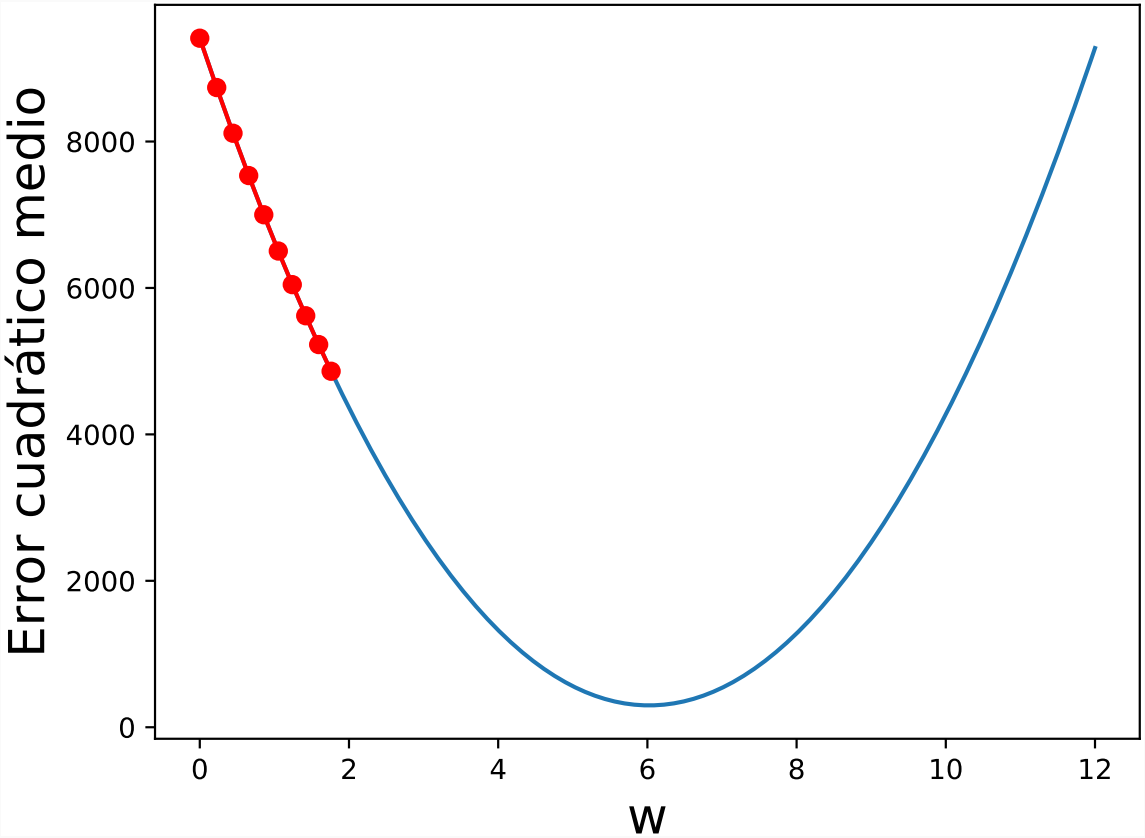
$$\mathbf{g}(\boldsymbol{\theta}) = \frac{d}{d\boldsymbol{\theta}} E = \mathbf{X}^T(\hat{\mathbf{y}} - \mathbf{y})$$

- Gradiente descendente estocástico: calcula gradiente con lote pequeño de datos

Entrenando el modelo: algoritmo del gradiente descendente



Sensibilidad a coeficiente de aprendizaje α



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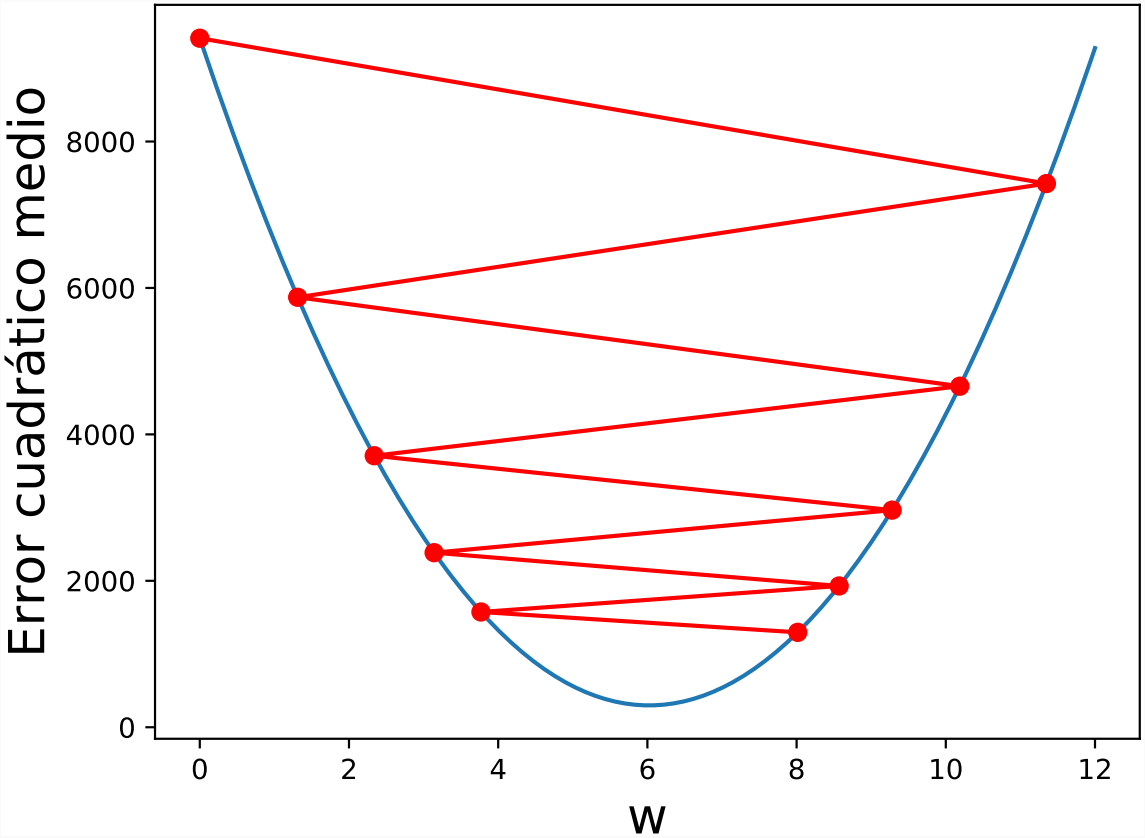


Problemas no lineales

- ¿Cómo modelamos una computer XOR?

x_1	x_2	XOR
0	0	0
0	1	1
1	0	1
1	1	0

Problemas no lineales

- ¿Cómo modelamos una computer XOR?

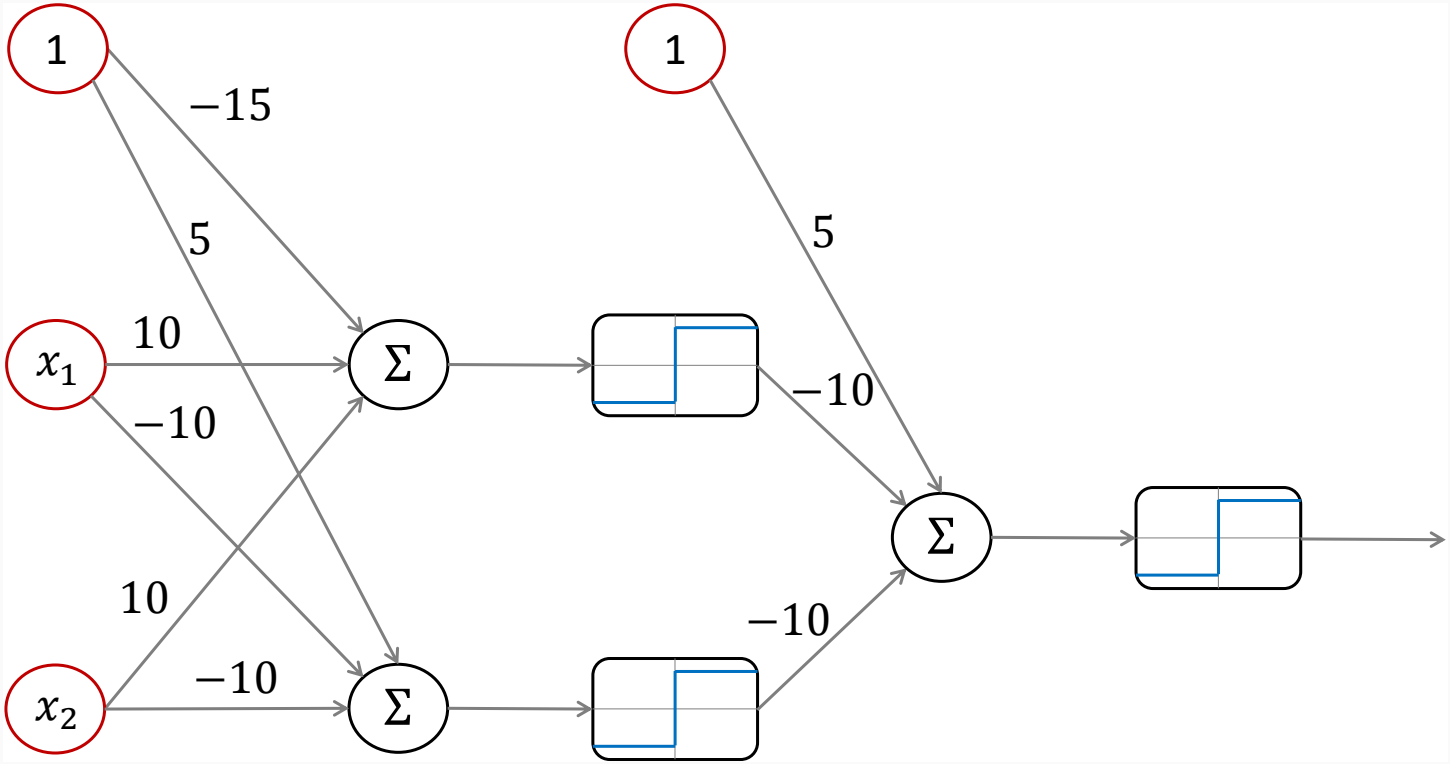
x_1	x_2	XOR
0	0	0
0	1	1
1	0	1
1	1	0

- Minsky y Papert demostraron que era imposible aprender la XOR con perceptrones

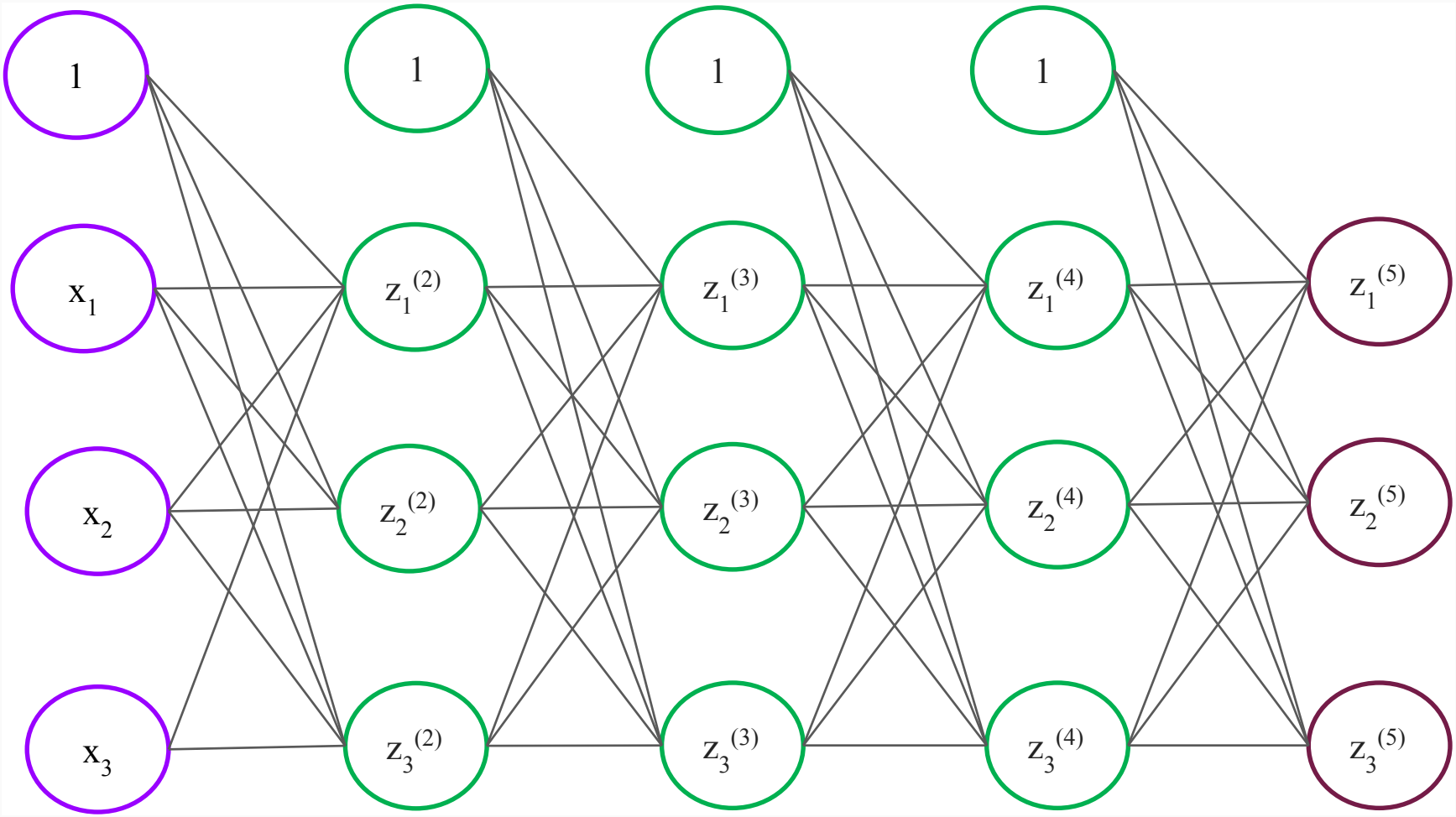
Múltiples capas

- Podemos usar la fórmula

$$x_1 \oplus x_2 = (x_1 \vee x_2) \wedge \neg(x_1 \wedge x_2)$$



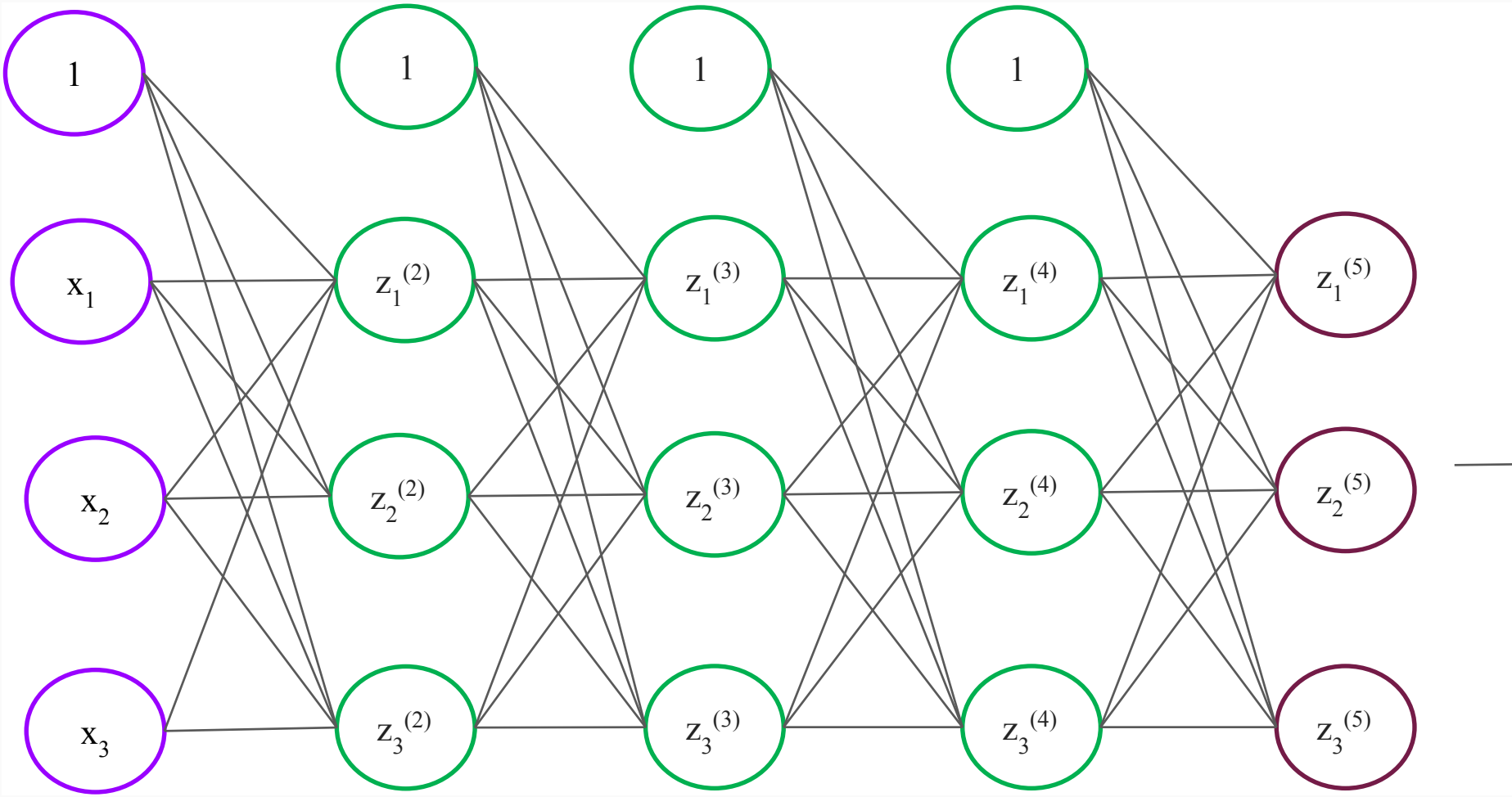
Red neuronal densa



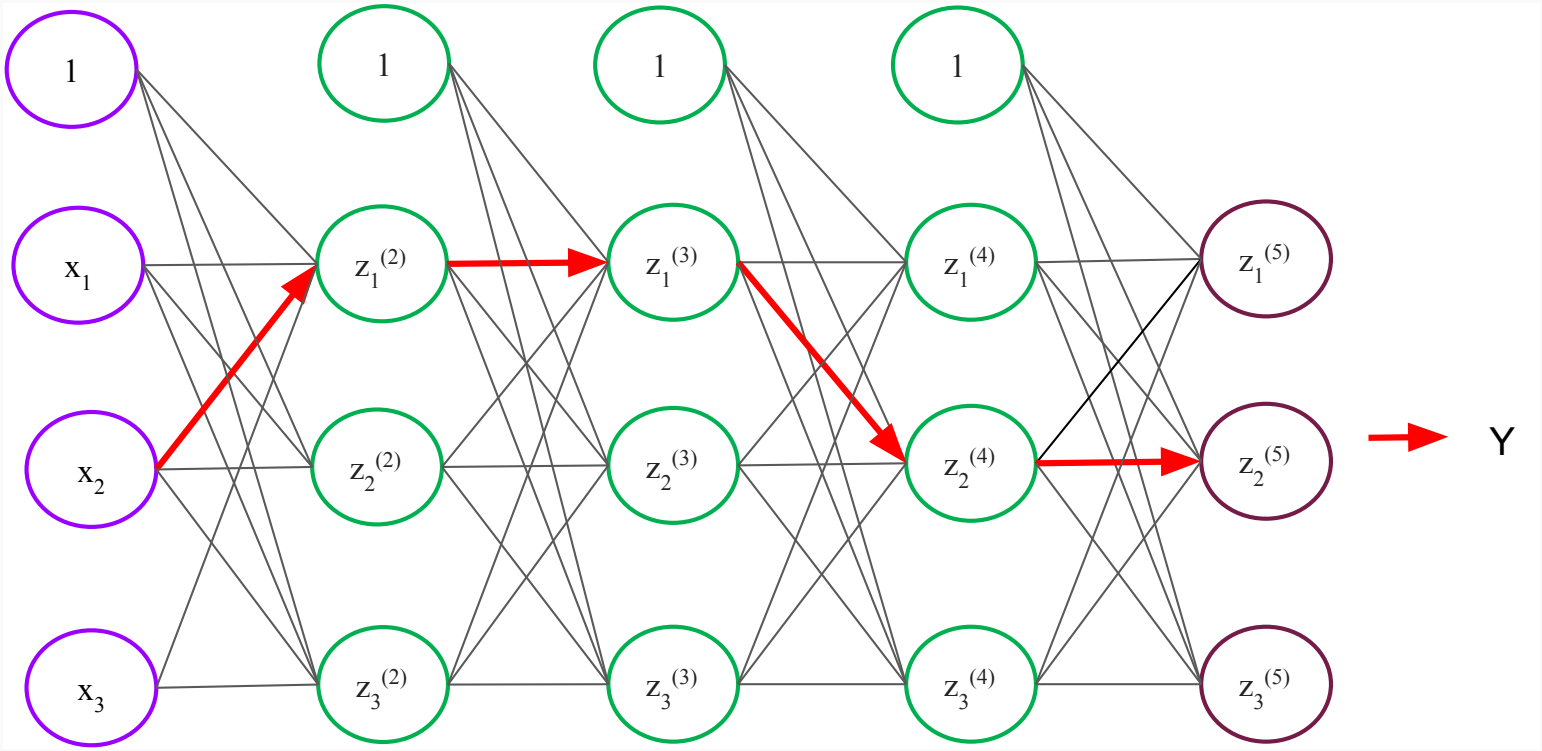
Pasos en la retro-propagación de errores

1. Propagamos hacia adelante las entradas para generar salidas
2. Calculamos error usando las salidas generadas y las salidas deseadas
3. Retro-propagamos error de salida hacia atrás

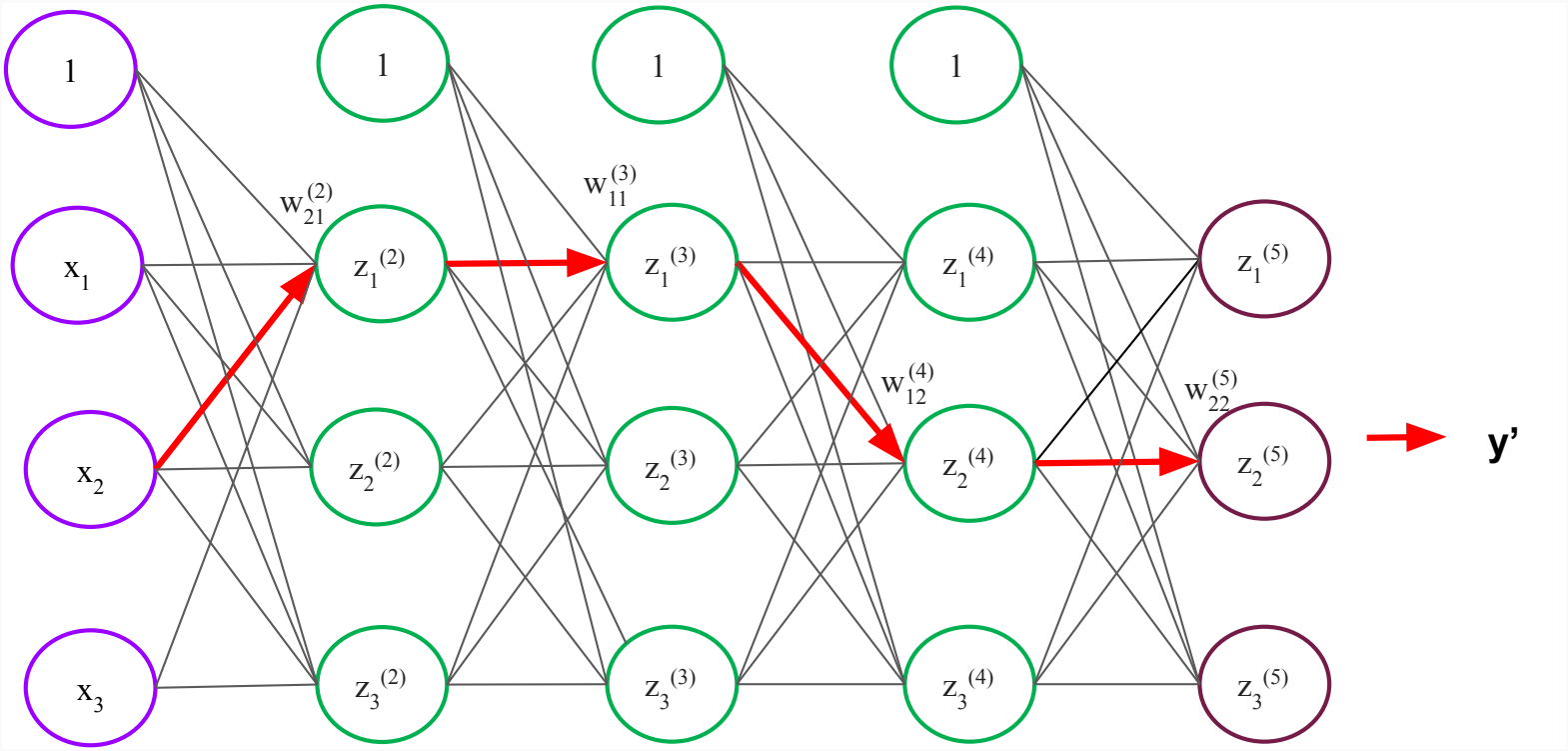
Propagación hacia adelante



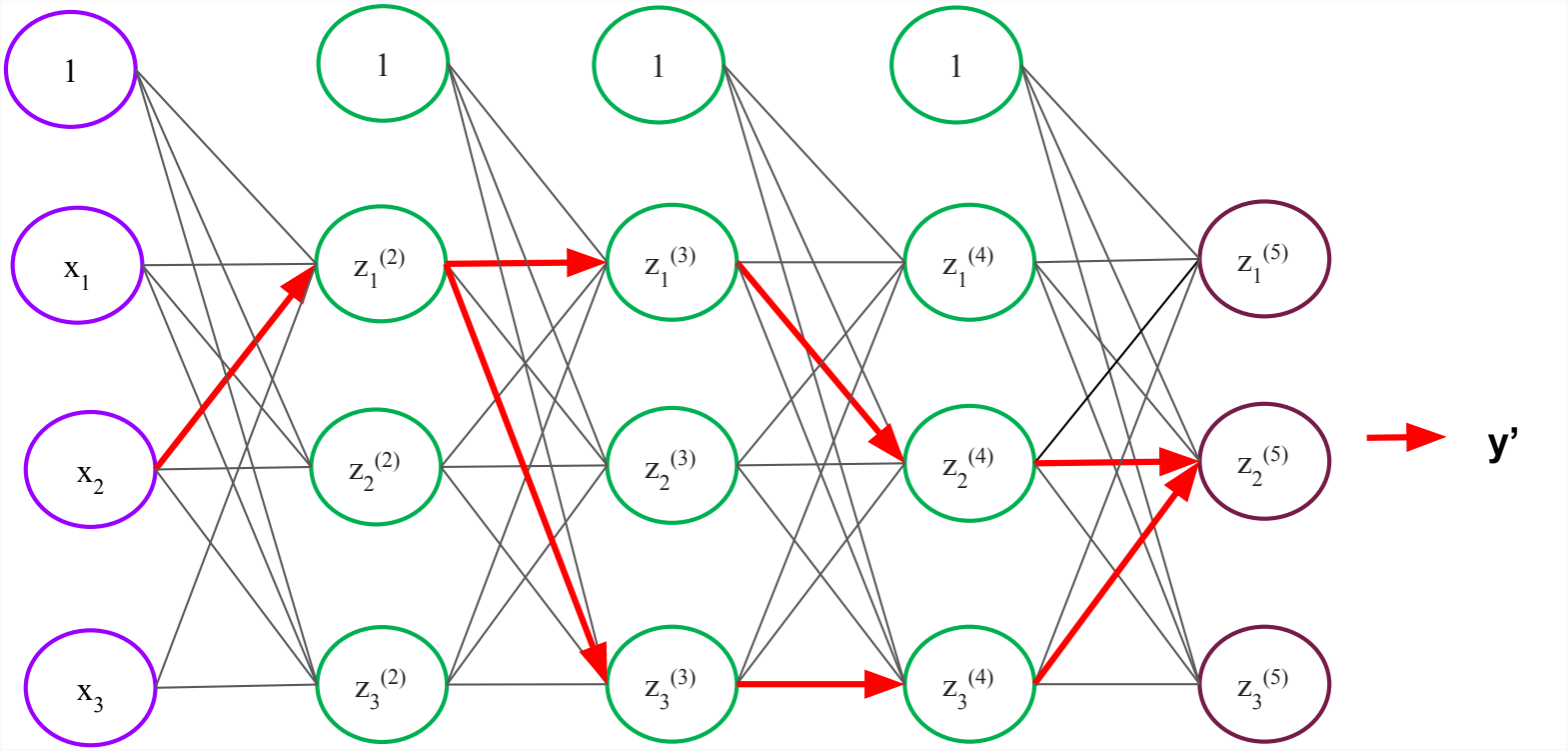
Entrenando por gradiente descendente



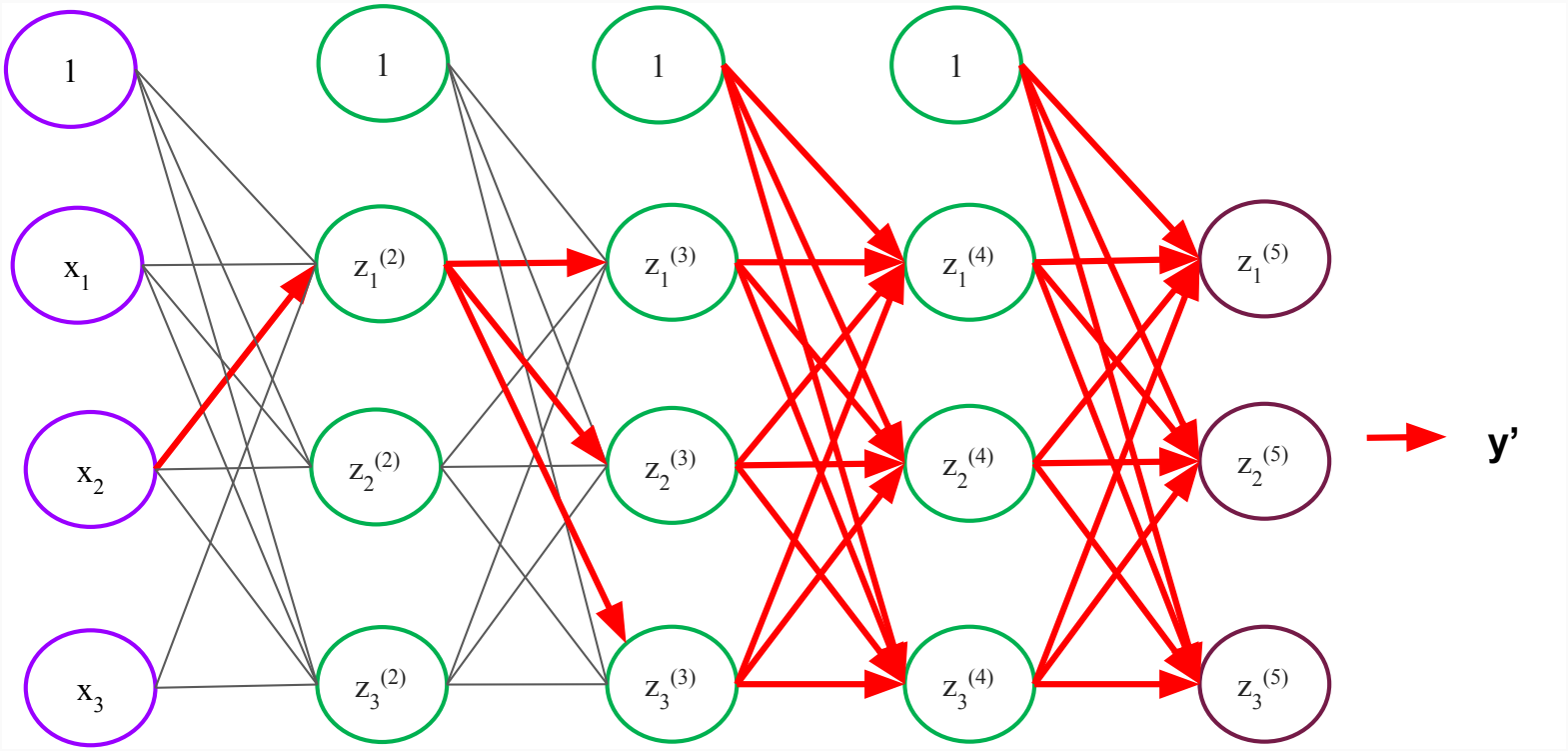
Entrenando por gradiente descendente



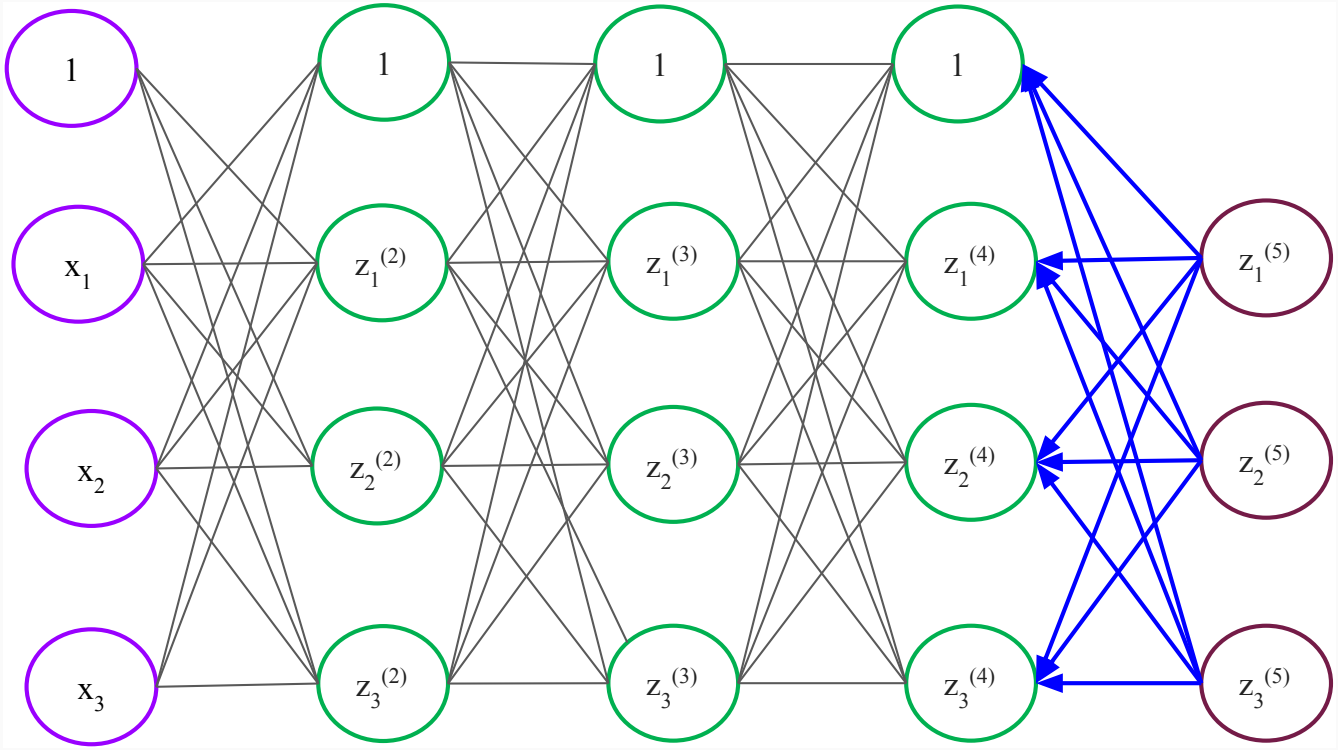
Entrenando por gradiente descendente



Entrenando por gradiente descendente



Algoritmo de retropropagación



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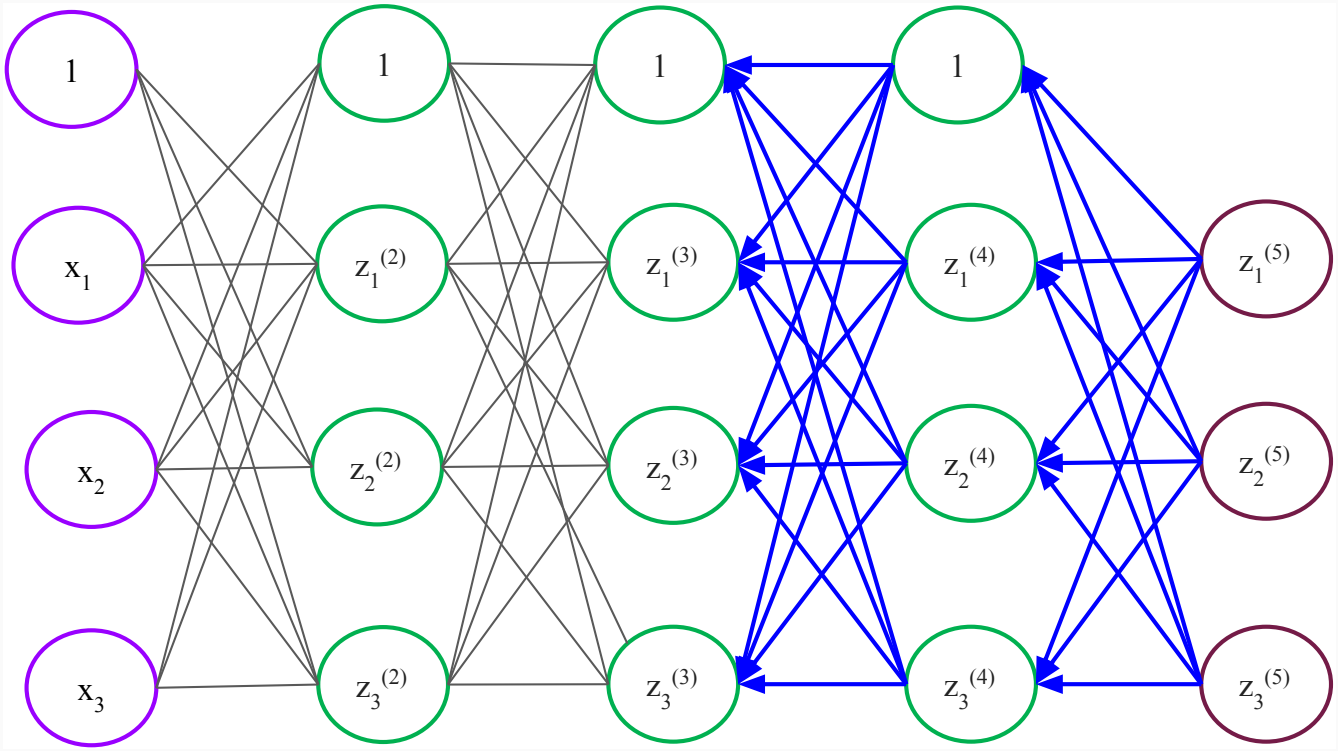

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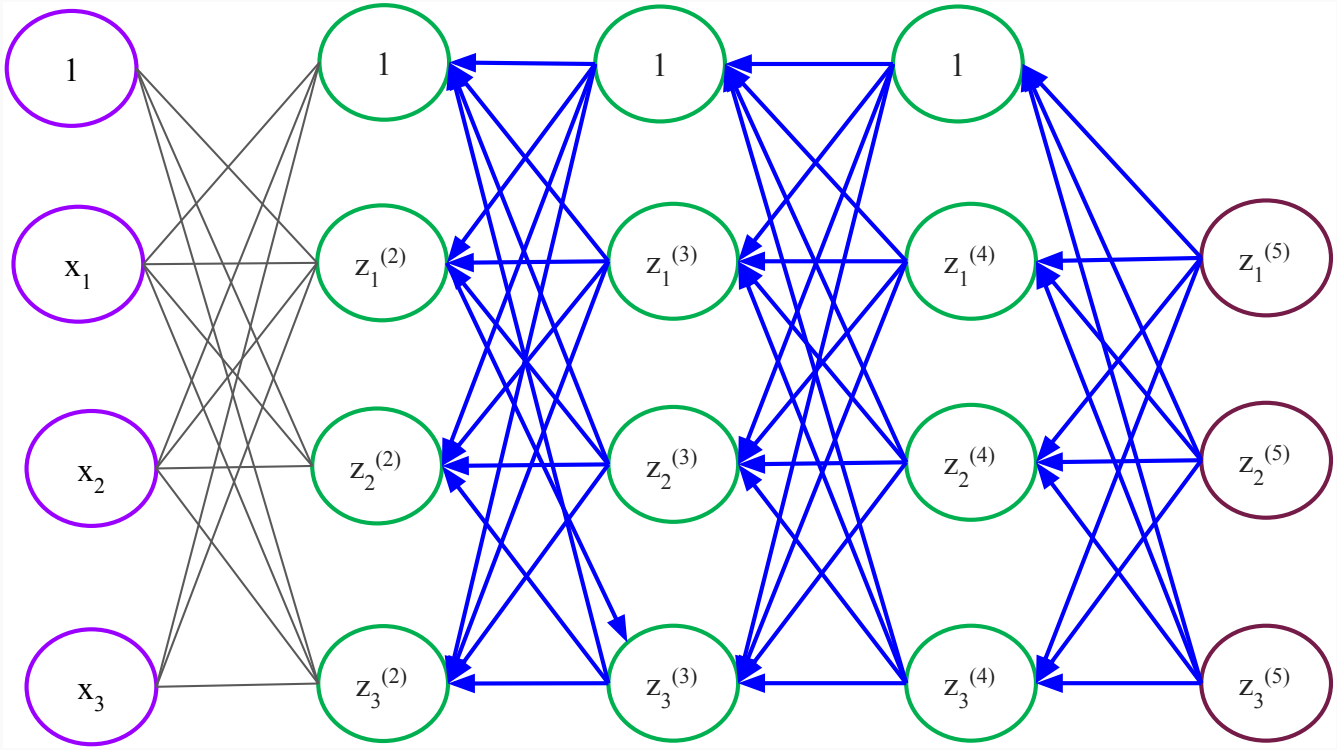

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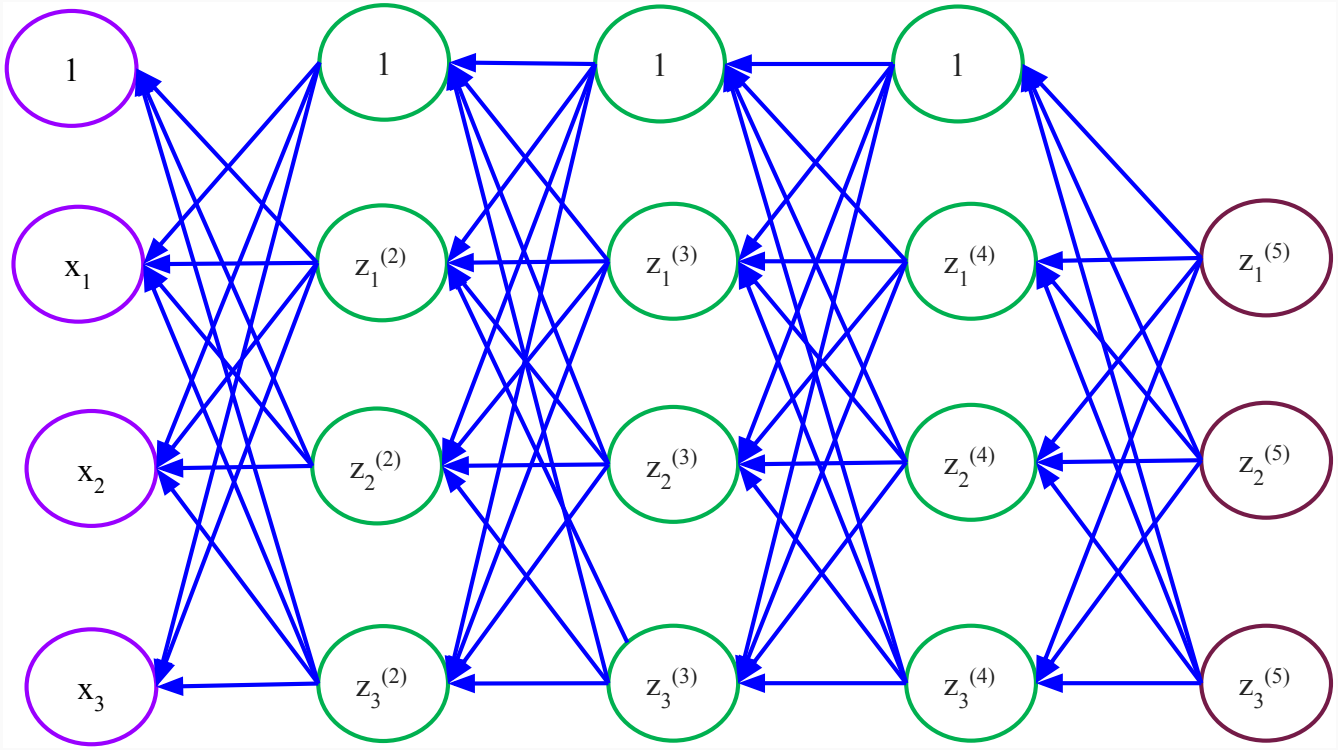


Propagación hacia adelante

- Propagamos hacia adelante las entradas para generar salidas

$$
\begin{aligned}
a^{(1)} &= x \\
z^{(2)} &= W^{(1)} \cdot a^{(1)} \\
a^{(2)} &= \phi(z^{(2)}) \\
z^{(3)} &= W^{(2)} \cdot a^{(2)} \\
a^{(3)} &= \phi(z^{(3)})
\end{aligned}
$$

Cálculo de error

- Calculamos error usando las salidas generadas y las salidas deseadas. Por ejemplo:

$$E \triangleq \sum \frac{1}{2}(y - \hat{y})^2$$

Retro-propagación de errores (1)

- Calculamos el gradiente del error con respecto a $W^(2)$ de la siguiente forma

$$\begin{aligned}
\frac{\partial E}{\partial W^{(2)}} &= \frac{\partial \sum \frac{1}{2}(y-\hat{y})^2}{\partial W^{(2)}} \\
&= \frac{\sum \partial \frac{1}{2}(y-\hat{y})^2}{\partial W^{(2)}} \\
\frac{\partial \frac{1}{2}(y-\hat{y})^2}{\partial W^{(2)}} &= (y-\hat{y}) \cdot \left(-\frac{\partial \hat{y}}{\partial W^{(2)}} \right) \\
&= (y-\hat{y}) \cdot \left(-\frac{\partial \hat{y}}{\partial z^{(3)}} \cdot \frac{\partial z^{(3)}}{\partial W^{(2)}} \right) \\
&= \underbrace{-(y-\hat{y}) \cdot \frac{\partial \hat{y}}{\partial z^{(3)}}}_{\delta^{(3)}} \cdot a^{(2)}
\end{aligned}$$

Retro-propagación de errores (2)

- Calculamos el gradiente del error con respecto a $W^(1)$ de la siguiente forma

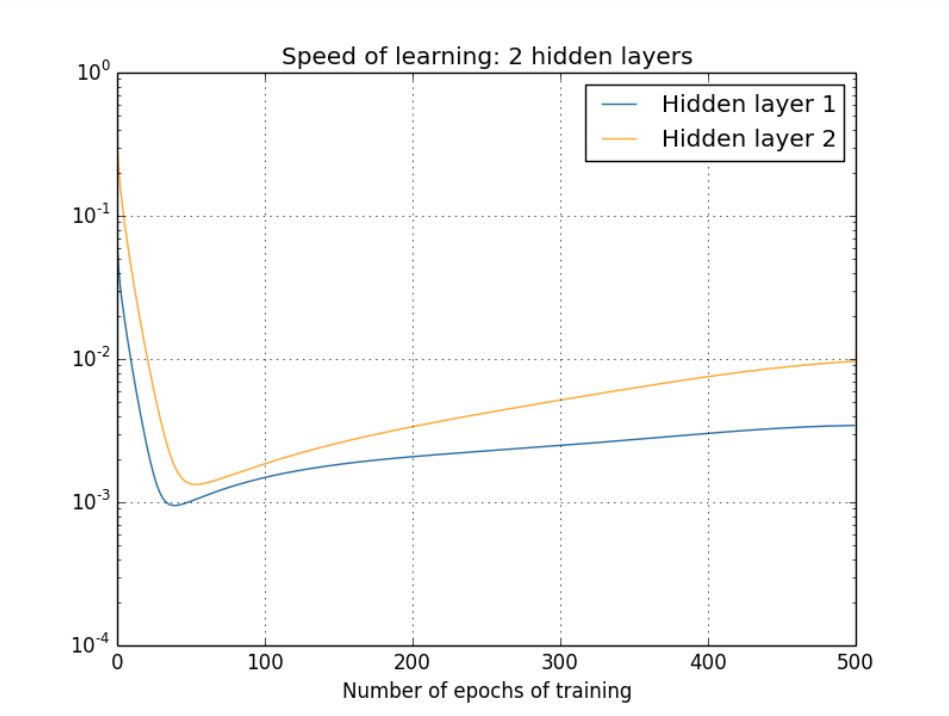
$$\begin{aligned}
\frac{\partial E}{\partial W^{(1)}} &= \frac{\partial \sum \frac{1}{2}(y-\hat{y})^2}{\partial W^{(1)}} \\
&= \frac{\sum \partial \frac{1}{2}(y-\hat{y})^2}{\partial W^{(1)}} \\
\frac{\partial \frac{1}{2}(y-\hat{y})^2}{\partial W^{(1)}} &= (y-\hat{y})\left(-\frac{\partial \hat{y}}{\partial W^{(1)}}\right) \\
&= (y-\hat{y})\left(-\frac{\partial \hat{y}}{\partial z^{(3)}} \cdot \frac{\partial z^{(3)}}{\partial W^{(1)}}\right) \\
&= \delta^{(3)} \cdot \frac{\partial z^{(3)}}{\partial W(1)}
\end{aligned}$$

Retro-propagación de errores (3)

$$
\begin{aligned}
&= \delta^{(3)} \cdot \left(\frac{\partial z^{(3)}}{\partial a^{(2)}} \cdot \frac{\partial a^{(2)}}{\partial W^{(1)}} \right) \\
&= \delta^{(3)} \cdot W^{(2)} \cdot \left(\frac{\partial a^{(2)}}{\partial W^{(1)}} \right) \\
&= \delta^{(3)} \cdot W^{(2)} \cdot \left(\frac{\partial a^{(2)}}{\partial z^{(2)}} \cdot \frac{\partial z^{(2)}}{\partial W^{(1)}} \right) \\
&= \delta^{(3)} \cdot W^{(2)} \cdot \frac{\partial a^{(2)}}{\partial z^{(2)}} \cdot x
\end{aligned}
$$

El problema del desvanecimiento del gradiente

- Gradientes de primeras capas se vuelven muy pequeños si la red es muy profunda
- Muy lento actualizar pesos de estas capas



Tomado de `http://neuralnetworksanddeeplearning.com/chap5.html`

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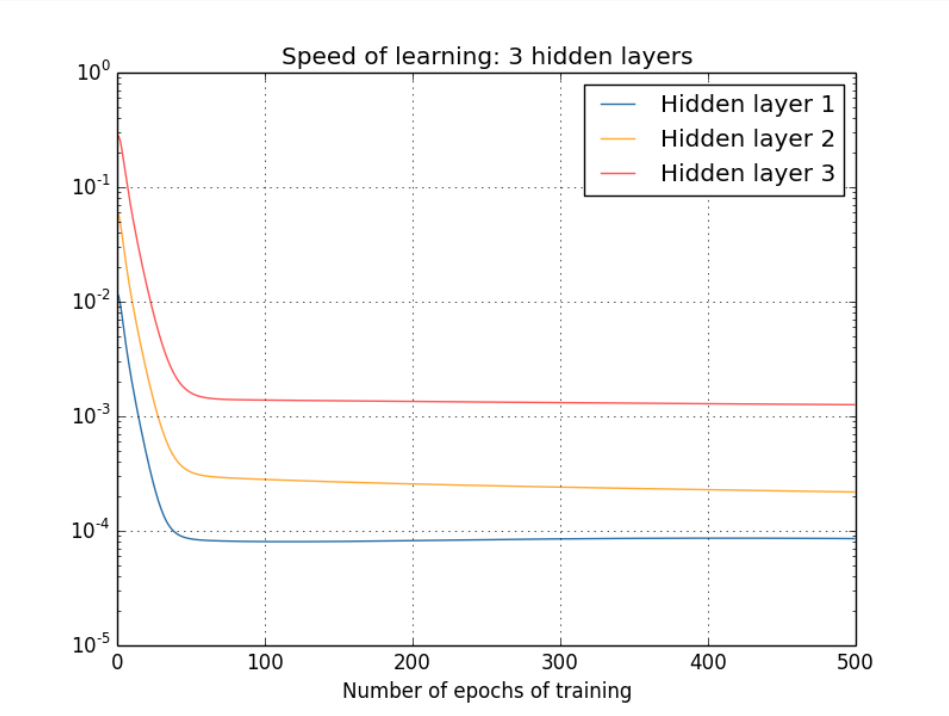

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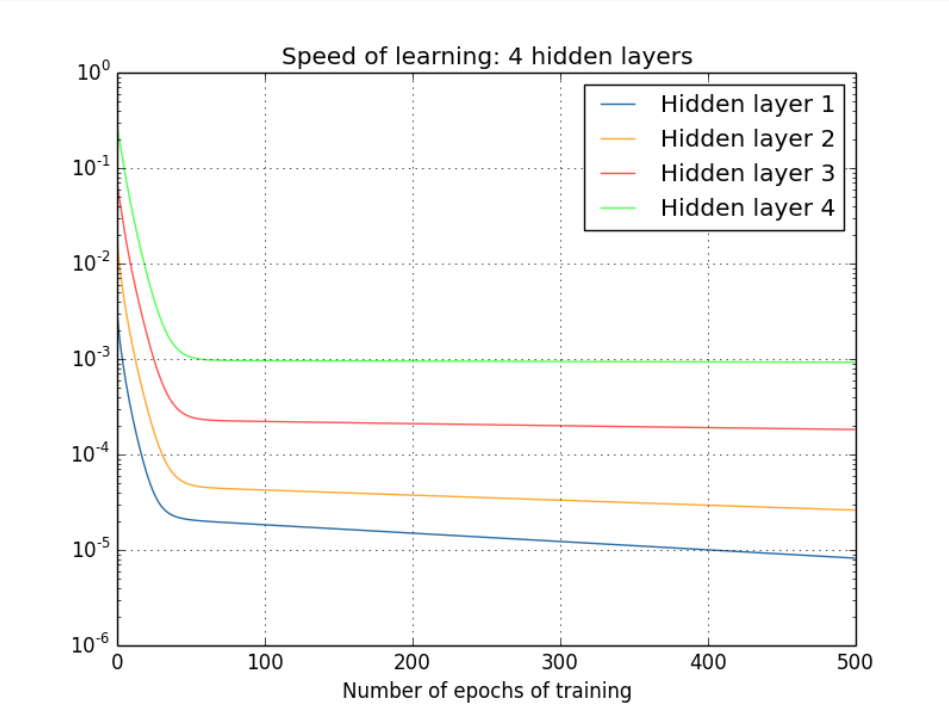
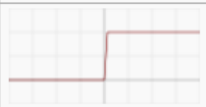
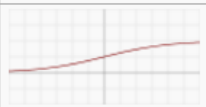
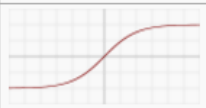
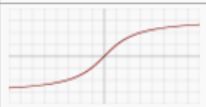
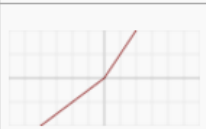
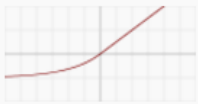
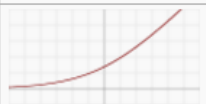


Tomado de `http://neuralnetworksanddeeplearning.com/chap5.html`

Funciones de activación

Name	Plot	Equation	Derivative
Identity		$f(x) = x$	$f'(x) = 1$
Binary step		$f(x) = \begin{cases} 0 & \text{for } x < 0 \\ 1 & \text{for } x \geq 0 \end{cases}$	$f'(x) = \begin{cases} 0 & \text{for } x \neq 0 \\ ? & \text{for } x = 0 \end{cases}$
Logistic (a.k.a Soft step)		$f(x) = \frac{1}{1+e^{-x}}$	$f'(x) = f(x)(1-f(x))$
TanH		$f(x) = \tanh(x) = \frac{2}{1+e^{-2x}} - 1$	$f'(x) = 1 - f(x)^2$
ArcTan		$f(x) = \tan^{-1}(x)$	$f'(x) = \frac{1}{x^2+1}$
Rectified Linear Unit (ReLU)		$f(x) = \begin{cases} 0 & \text{for } x < 0 \\ x & \text{for } x \geq 0 \end{cases}$	$f'(x) = \begin{cases} 0 & \text{for } x < 0 \\ 1 & \text{for } x \geq 0 \end{cases}$
Parameteric Rectified Linear Unit (PReLU) [2]		$f(x) = \begin{cases} \alpha x & \text{for } x < 0 \\ x & \text{for } x \geq 0 \end{cases}$	$f'(x) = \begin{cases} \alpha & \text{for } x < 0 \\ 1 & \text{for } x \geq 0 \end{cases}$
Exponential Linear Unit (ELU) [3]		$f(x) = \begin{cases} \alpha(e^x - 1) & \text{for } x < 0 \\ x & \text{for } x \geq 0 \end{cases}$	$f'(x) = \begin{cases} f(x) + \alpha & \text{for } x < 0 \\ 1 & \text{for } x \geq 0 \end{cases}$
SoftPlus		$f(x) = \log_e(1+e^x)$	$f'(x) = \frac{1}{1+e^{-x}}$

Tomado de Wikipedia (Activation Function)