Session 23

Bottom-up Parsing

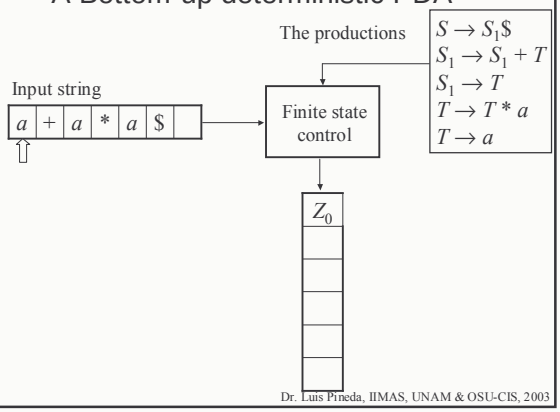
Deterministic bottom-up parser

- In the bottom-up simulation of a PDA, non-determinism appears when there is the choice between:
  - Shift a symbol onto the stack or reduce a string on top of the stack
  - If reduce, there may be more than one right-side of a production that matches a string in the top of the stack
- In a deterministic bottom-up parser we must be able to do the right choice by looking at the next symbol on the input tape!

Deterministic bottom-up parser

- Shift versus reduce:
  - There are combinations of top of the stack symbol and input symbol in which reduction is always appropriate, and shift is appropriate for all other combinations
- Appropriate reduction:
  - The one that reduces the longest possible string

A Bottom-up deterministic PDA



Deterministic bottom-up parser

The productions:

$S \to S_1\$$
$S_1 \to S_1 + T$
$S_1 \to T$
$T \to T * a$
$T \to a$

Shift moves:

Id	State	Input	Stack symbol	Move(s)
1	q	σ	X	$(q, \sigma Z_0)$
2	q	σ	T	$(q_1, \sigma T)$

For 1: σ is anything and X is Z_0, S_1, + or *

For 2: σ is an input symbol other than + or $

Deterministic bottom-up parser

$S \to S_1\$$
$S_1 \to S_1 + T$
$S_1 \to T$
$T \to T * a$
$T \to a$

Moves to reduce $S_1\$$ to S

Id	State	Input	Stack symbol	Move(s)
3	q	Λ	$	$(q_\$, \Lambda)$
4	$q_\$$	Λ	S_1	(q, S)

Deterministic bottom-up parser

$S \to S_1\$$
$S_1 \to S_1 + T$
$S_1 \to T$
$T \to T * a$
$T \to a$

Moves to reduce either a or $T* A$ to T

Id	State	Input	Stack symbol	Move(s)
5	q	Λ	a	$(q_{a,1}, \Lambda)$
6	$q_{a,1}$	Λ	*	$(q_{a,2}, \Lambda)$
7	$q_{a,2}$	Λ	T	(q, T)
8	$q_{a,1}$	Λ	X	(q, TX)

For 8: X is any stack symbol other than *

Deterministic bottom-up parser

$S \to S_1\$$
$S_1 \to S_1 + T$
$S_1 \to T$
$T \to T * a$
$T \to a$

Moves to reduce either $S_1 + T$ or T to S_1 and shift an input symbol

Id	State	Input	Stack symbol	Move(s)
9	q	σ	T	$(q_{T,\sigma}, \Lambda)$
10	$q_{T,\sigma}$	Λ	+	$(q'_{T,\sigma}, \Lambda)$
11	$q'_{T,\sigma}$	Λ	S_1	$(q, \sigma S_1)$
12	$q_{T,\sigma}$	Λ	X	$(q, \sigma S_1 X)$

For 8: X is either + or \$; X is stack symbol other than +

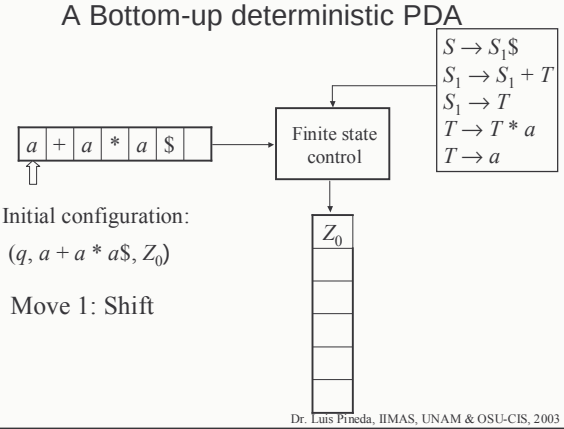
Deterministic bottom-up parser

$S \to S_1\$$
$S_1 \to S_1 + T$
$S_1 \to T$
$T \to T * a$
$T \to a$

Moves to accept

Id	State	Input	Stack symbol	Move(s)
13	q	Λ	S	(q_1, Λ)

A Bottom-up deterministic PDA

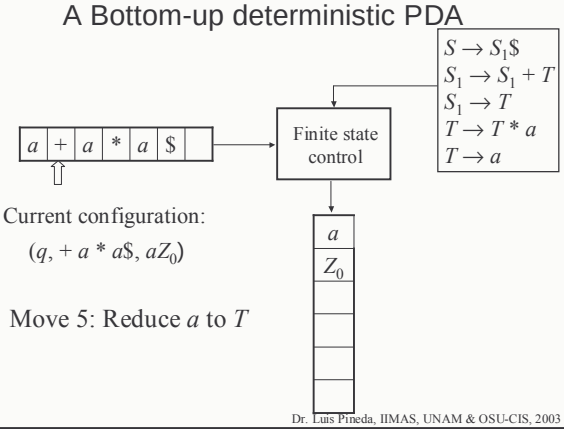


Initial configuration:

$(q, a + a * a\$, Z_0)$

Move 1: Shift

A Bottom-up deterministic PDA

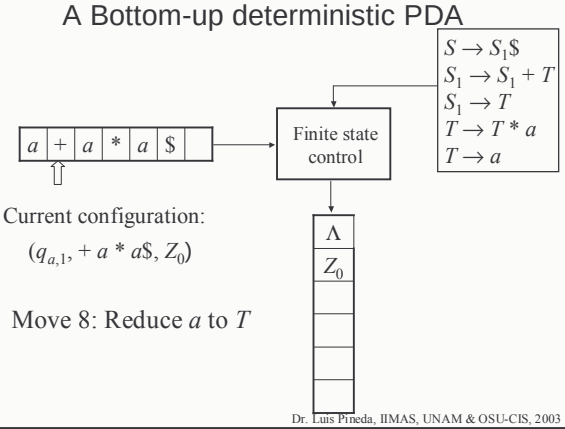


Current configuration:

$(q, + a * a\$, aZ_0)$

Move 5: Reduce a to T

A Bottom-up deterministic PDA

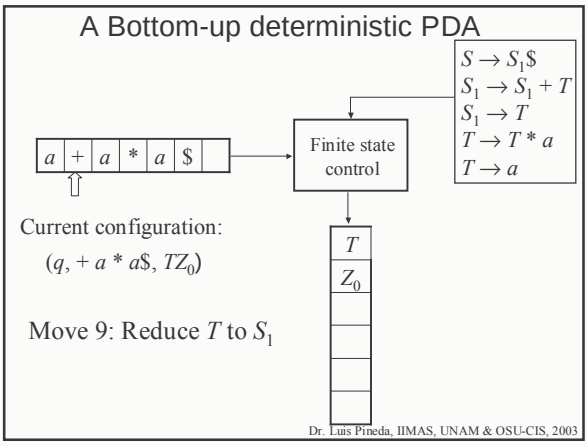


Current configuration:

$(q_{a,1}, + a * a\$, Z_0)$

Move 8: Reduce a to T

A Bottom-up deterministic PDA

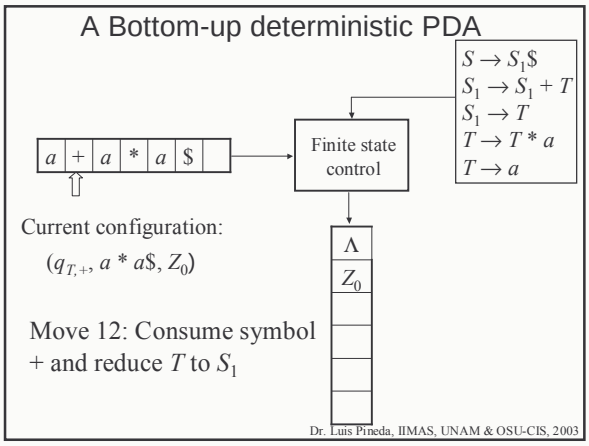


Current configuration:

$(q, + a * a\$, TZ_0)$

Move 9: Reduce T to S_1

A Bottom-up deterministic PDA

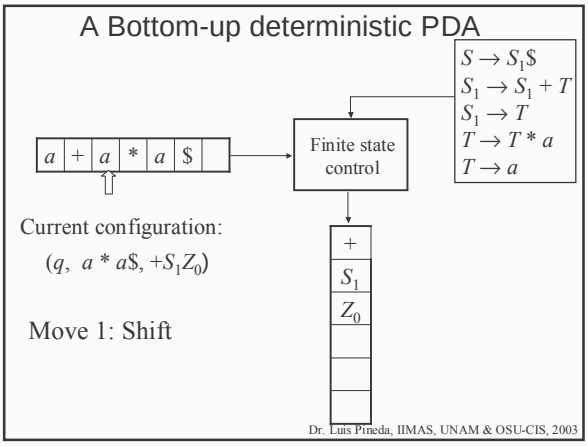


Current configuration:

$(q_{T,+}, a * a\$, Z_0)$

Move 12: Consume symbol + and reduce T to S_1

A Bottom-up deterministic PDA

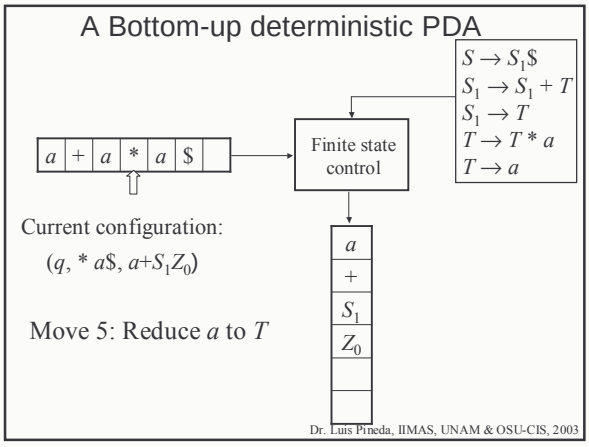


Current configuration:

$(q, \ a * a\$, +S_1Z_0)$

Move 1: Shift

A Bottom-up deterministic PDA

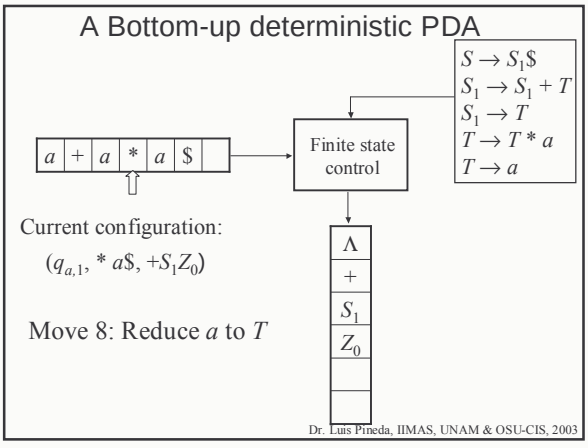


Current configuration:

$(q, * a\$, a+S_1Z_0)$

Move 5: Reduce a to T

A Bottom-up deterministic PDA

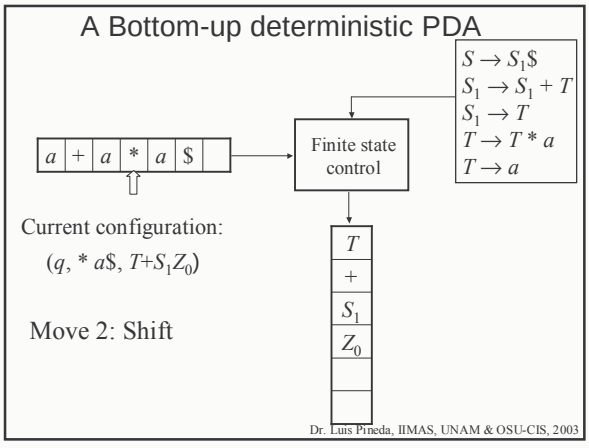


Current configuration:

$(q_{a,1}, * a\$, +S_1Z_0)$

Move 8: Reduce a to T

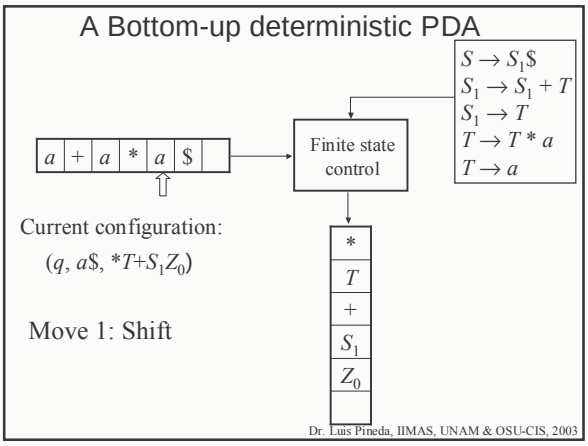
A Bottom-up deterministic PDA

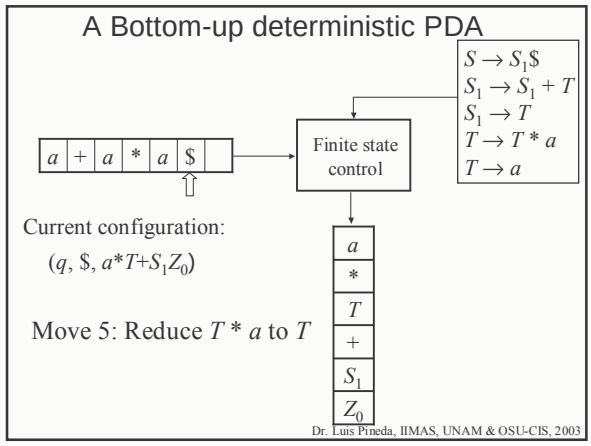


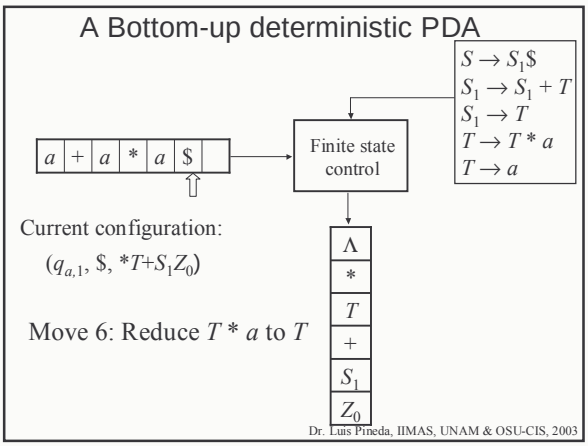
Current configuration:

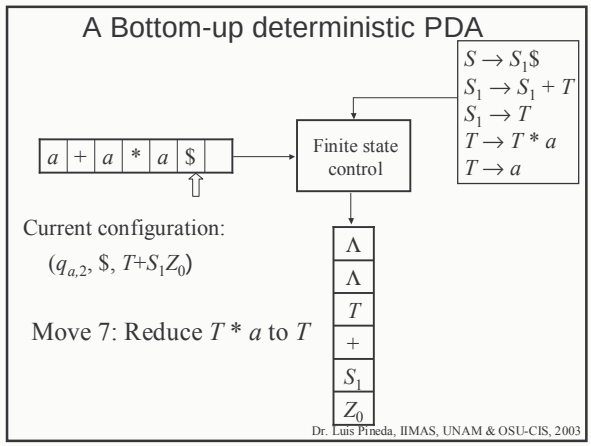
$(q, * a\$, T+S_1Z_0)$

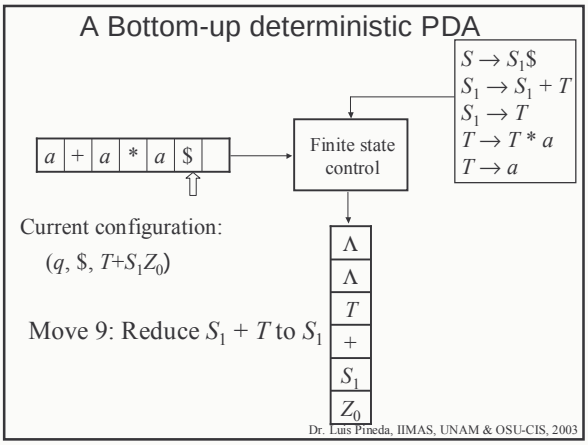
Move 2: Shift

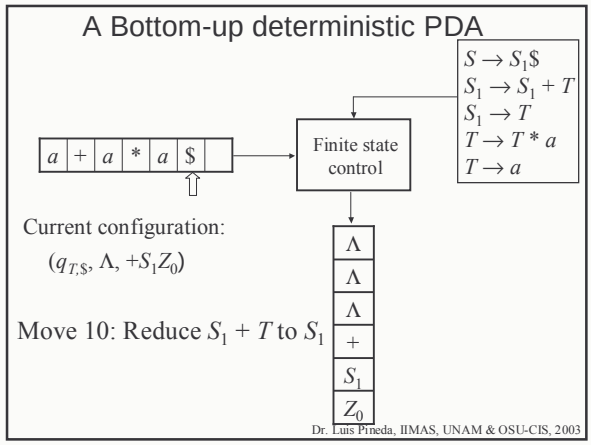

A Bottom-up deterministic PDA
$S \to S_1\$$
$S_1 \to S_1 + T$
$S_1 \to T$
$T \to T * a$
$T \to a$
a + a * a $
Finite state control
Current configuration:
$(q, a\$, *T+S_1Z_0)$
Move 1: Shift
*
T
+
S_1
Z_0
Dr. Luis Pineda, IIMAS, UNAM & OSU-CIS, 2003


A Bottom-up deterministic PDA
$S \to S_1\$$
$S_1 \to S_1 + T$
$S_1 \to T$
$T \to T * a$
$T \to a$
a + a * a $
Finite state control
Current configuration:
$(q, \$, a*T+S_1Z_0)$
Move 5: Reduce $T * a$ to T
a
*
T
+
S_1
Z_0
Dr. Luis Pineda, IIMAS, UNAM & OSU-CIS, 2003

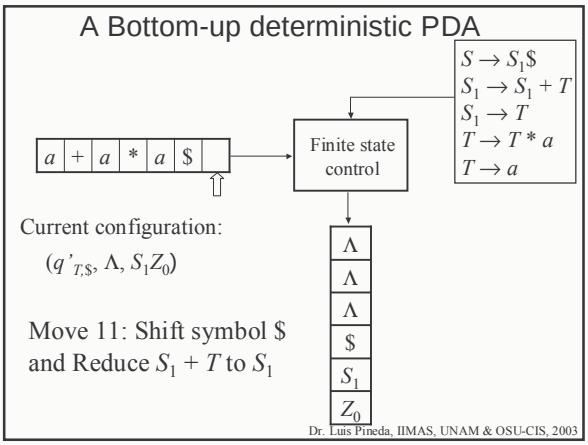

A Bottom-up deterministic PDA
$S \to S_1\$$
$S_1 \to S_1 + T$
$S_1 \to T$
$T \to T * a$
$T \to a$
a + a * a $
Finite state control
Current configuration:
$(q_{a,1}, \$, *T+S_1Z_0)$
Move 6: Reduce $T * a$ to T
Λ
*
T
+
S_1
Z_0
Dr. Luis Pineda, IIMAS, UNAM & OSU-CIS, 2003


A Bottom-up deterministic PDA
$S \to S_1\$$
$S_1 \to S_1 + T$
$S_1 \to T$
$T \to T * a$
$T \to a$
a + a * a $
Finite state control
Current configuration:
$(q_{a,2}, \$, T+S_1Z_0)$
Move 7: Reduce $T * a$ to T
Λ
Λ
T
+
S_1
Z_0
Dr. Luis Pineda, IIMAS, UNAM & OSU-CIS, 2003


A Bottom-up deterministic PDA
$S \to S_1\$$
$S_1 \to S_1 + T$
$S_1 \to T$
$T \to T * a$
$T \to a$
a + a * a $
Finite state control
Current configuration:
$(q, \$, T+S_1Z_0)$
Move 9: Reduce $S_1 + T$ to S_1
Λ
Λ
T
+
S_1
Z_0
Dr. Luis Pineda, IIMAS, UNAM & OSU-CIS, 2003


A Bottom-up deterministic PDA
$S \to S_1\$$
$S_1 \to S_1 + T$
$S_1 \to T$
$T \to T * a$
$T \to a$
a + a * a $
Finite state control
Current configuration:
$(q_{T,\$}, \Lambda, +S_1Z_0)$
Move 10: Reduce $S_1 + T$ to S_1
Λ
Λ
Λ
+
S_1
Z_0
Dr. Luis Pineda, IIMAS, UNAM & OSU-CIS, 2003

A Bottom-up deterministic PDA

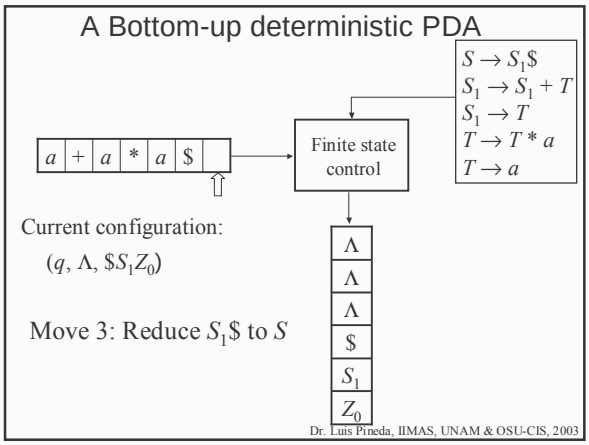


Current configuration:

$(q'_{T,\$}, \Lambda, S_1Z_0)$

Move 11: Shift symbol \$ and Reduce $S_1 + T$ to S_1

A Bottom-up deterministic PDA

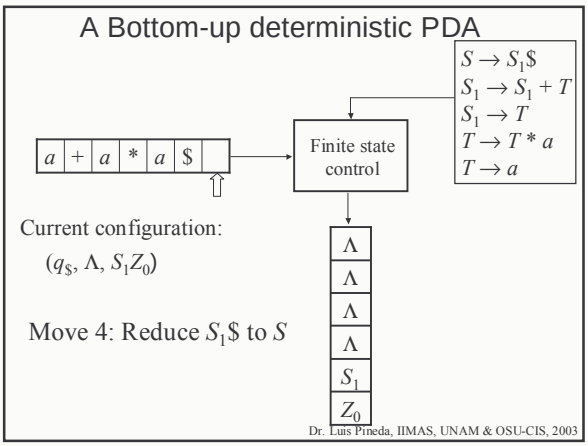


Current configuration:

$(q, \Lambda, \$S_1Z_0)$

Move 3: Reduce $S_1\$$ to S

A Bottom-up deterministic PDA

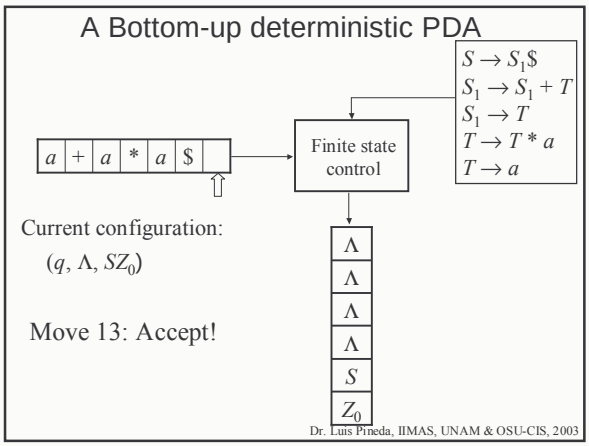


Current configuration:

$(q_\$, \Lambda, S_1Z_0)$

Move 4: Reduce $S_1\$$ to S

A Bottom-up deterministic PDA

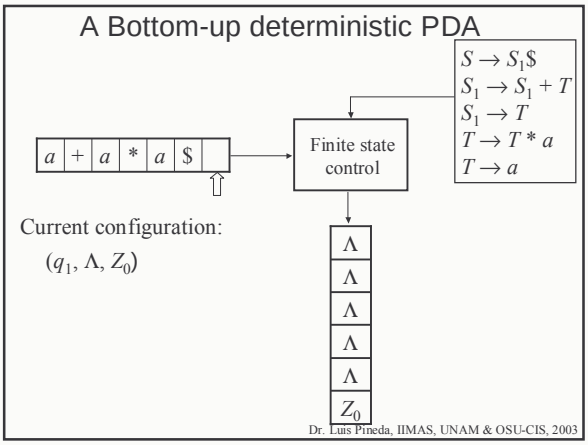


Current configuration:

(q, Λ, SZ_0)

Move 13: Accept!

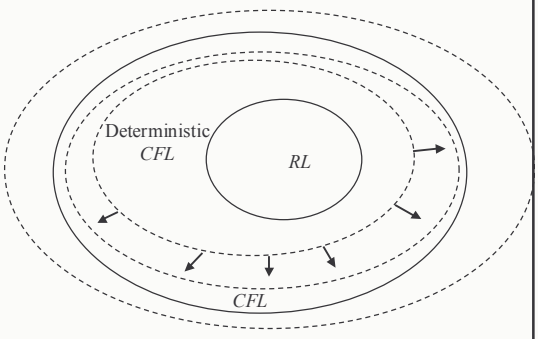
A Bottom-up deterministic PDA



Current configuration:

(q_1, Λ, Z_0)

Deterministic parsing and DCFL



- Deterministic parsing extends the class of DCFL