Session 8

Nondeterministic Finite Automata

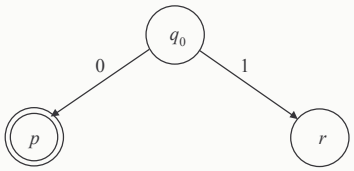
Nondeterministic FA

- Motivation
- Concept of non-determinism
- Definition of NFA
- Extended Transition Function for NFA
- Acceptance by a NFA

Motivation

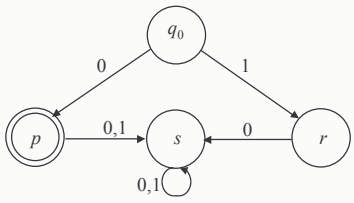
- Design a DFA accepting: **{11,110}*{0}**

- Let's proceed one symbol at a time: {11,110}*{0}



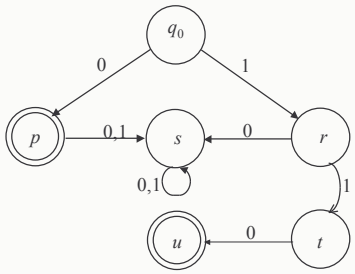
- The null string Λ is not in L, and q_0 is not an accepting state
- But 0 is in L, and it should take us to an accepting state
- 1 is initial symbol in both 11 and 110, the constituent words of the prefix of strings in the language (before the last 0).

- Design a DFA accepting: {11,110}*{0}



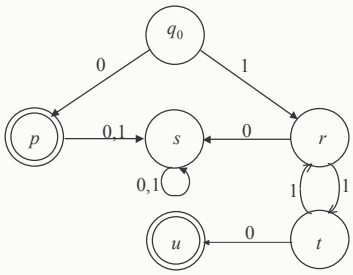
- L does not contain:
  - Strings starting with 0
  - Strings starting with 10
- We have a sink or dead state!

- Design a DFA accepting: {11,110}*{0}



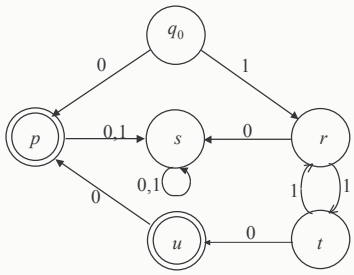
- In state r we have seen one 1, and we need 10

- Design a DFA accepting: $\{11,110\}^*\{0\}$



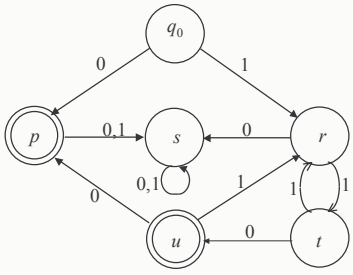
- But we also need to consider repetitions of 11
- The state r: having seen an odd sequence of 1's

- Design a DFA accepting: $\{11,110\}^*\{0\}$



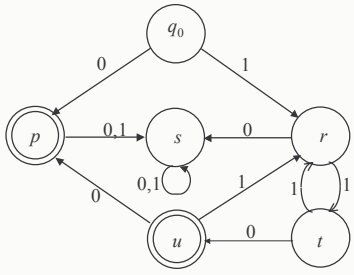
- But also, after 1, we can accept with 100

- Design a DFA accepting: $\{11,110\}^*\{0\}$



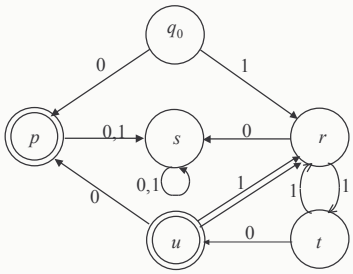
- But, the string 110 before the last 0 can also appear n times!

- Design a DFA accepting: $\{11,110\}^*\{0\}$



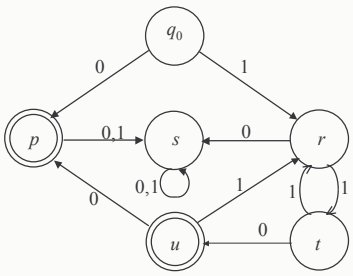
- State r also represents having seen the first 1 after 110, the first time

- Design a DFA accepting: $\{11,110\}^*\{0\}$



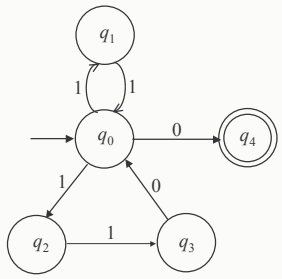
- State r also represents having seen the first 1 after 110, or after n number of times!

- Design a DFA accepting: $\{11,110\}^*\{0\}$



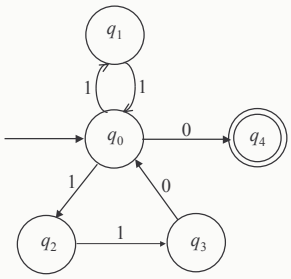
- For more complicated *RE*
  - Finding the equivalent FA can be quite laborious!
  - Making sure it is right: very complicated!

- But what about this, for accepting: **{11, 110}*{0}**



- We can read the expression in the diagram directly!

- But what about this, for accepting: **{11, 110}*{0}**



- We can read the expression in the diagram directly!

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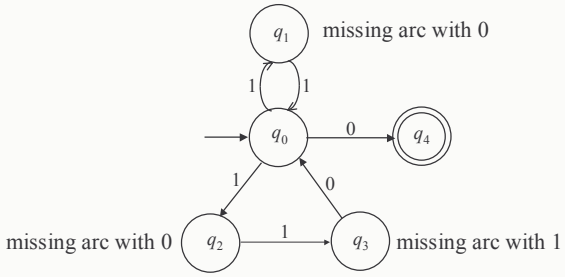
- But what about this, for accepting: **{11,110}*{0}**



- But we have states with no transitions on all symbols of Σ:
  No problem: we assume there is a non-accepting *sink state*

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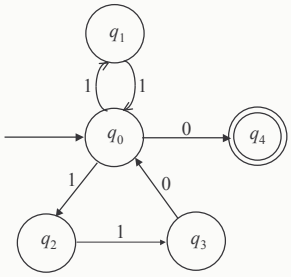
- But what about this, for accepting: **{11,110}*{0}**



- A state has two different next states on the same symbol!
- This FA is not deterministic: NFA

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Concept of non-determinism

- An FA is not deterministic if there is more than one next state for any state on the same input symbol
- We can think of it in several ways:
  - As *n* DFAs running in parallel, each one taking care of a given path
  - As a automata that "guesses" the next state when it has a choice
  - As an abstract specification of a computation, regardless the actual details of the algorithm or machine that performs the computation
  - Non-determinism allows to think disjunctively about FA!

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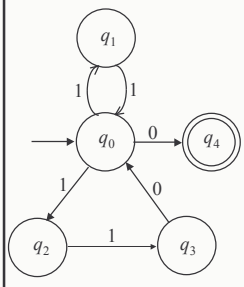
- Let's compute 11110:



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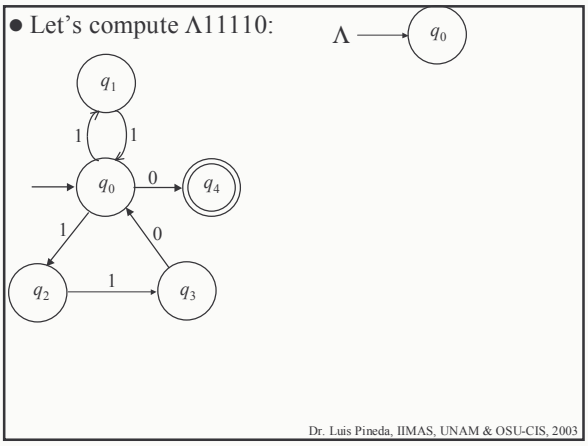
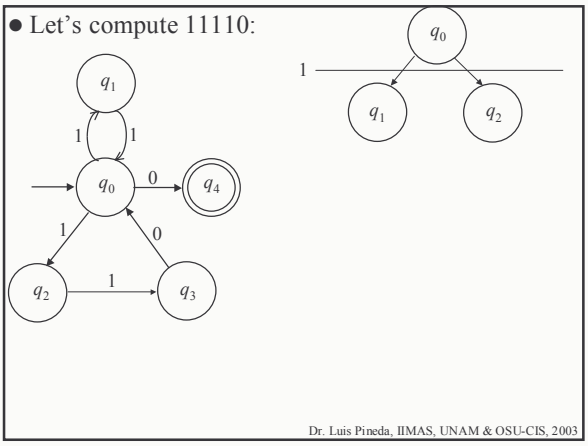
● Let's compute Λ11110:



● Let's compute 11110:



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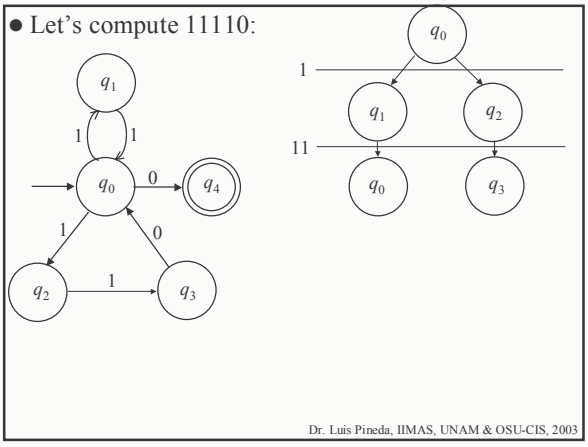
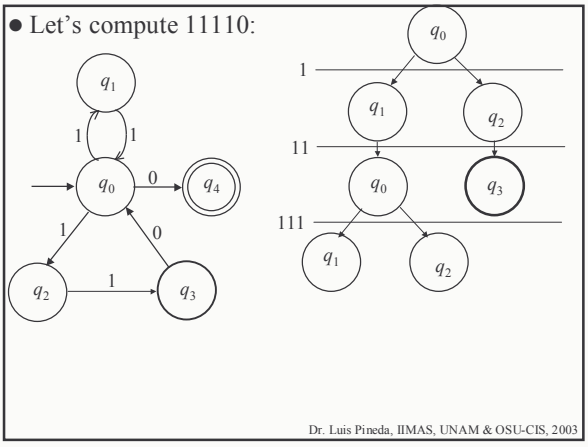
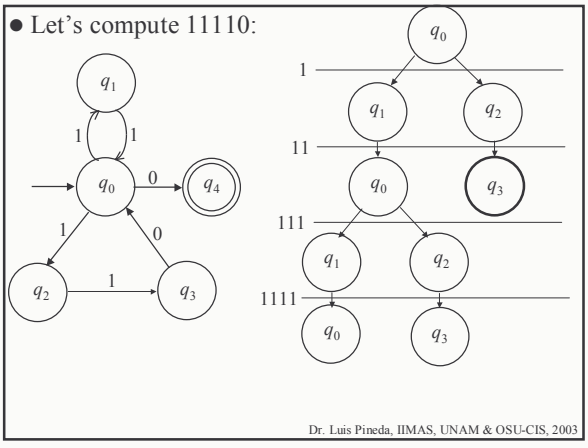


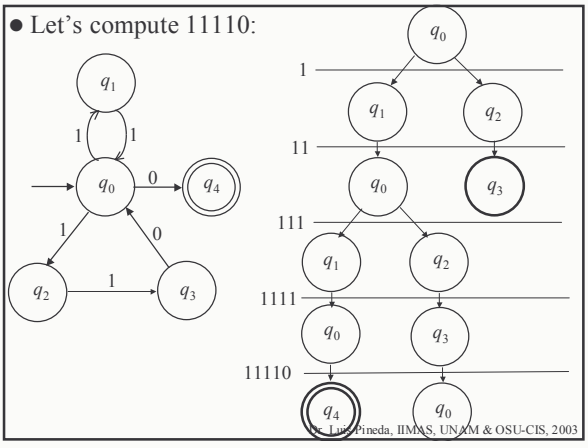
● Let's compute 11110:



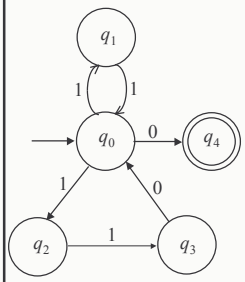
● Let's compute 11110:



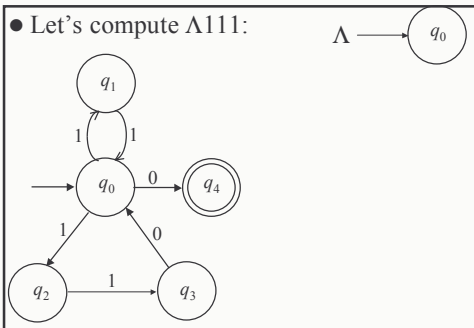
● Let's compute 11110:



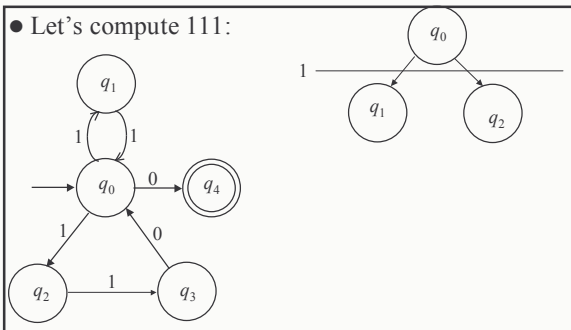
- But suppose we want 111, which is not in **{11,110}*{0}**



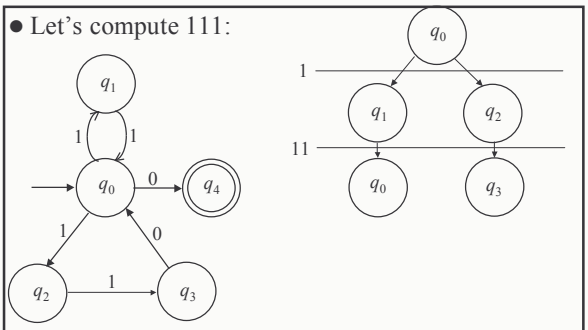
- Let's compute Λ111:



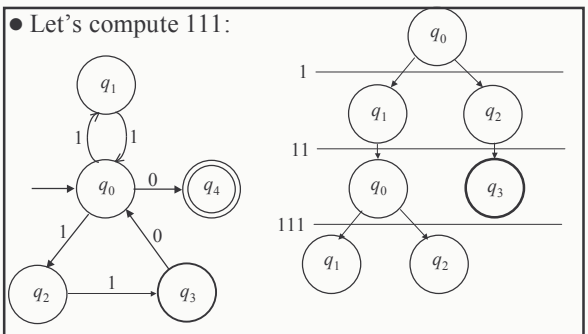
- Let's compute 111:



- Let's compute 111:



- Let's compute 111:



- The string is rejected: it is finished, and we are in no accepting state

Nondeterminism and abstraction

- Computations on tree-structures are non-deterministic
- Search strategies serialize the non-deterministic paths
- A declarative specification allow us to see whether a given condition is satisfied, independently of a concrete computation
- Non-determinism allows us to express disjunctive abstraction
- Provides the abstraction import of the union operator in *RE*

Definition of NFA

- A non-deterministic *finite Automaton* (NFA) is a 5-tuple $M = (Q, \Sigma, q_0, A, \delta)$, where
  - Q is a finite set (of states)
  - Σ is a finite alphabet
  - $q_0 \in Q$ (the initial state)
  - $A \subseteq Q$ (the set of accepting states)
  - A transition function:

    $\delta: Q \times \Sigma \rightarrow 2^Q$
- The only difference between DAF y NFA is the type δ

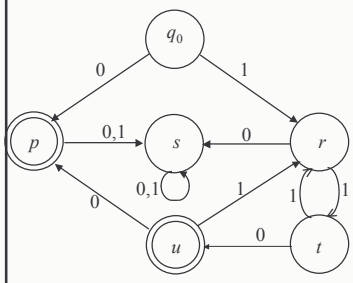
Transition function

- Transition function for DFA:

  $\delta: Q \times \Sigma \rightarrow Q$
- Transition function for NFA:

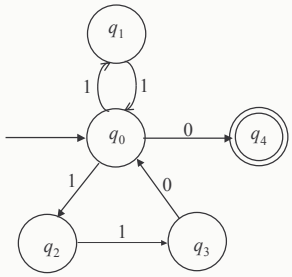
  $\delta: Q \times \Sigma \rightarrow 2^Q$
- The type of the range of δ is a set of states!

Transition Table DFA



	0	1
q_0	p	r
p	s	s
s	s	s
r	s	t
t	u	r
u	p	r

Transition Table NFA



	0	1
$\rightarrow q_0$	$\{q_4\}$	$\{q_1, q_2\}$
q_1	ϕ	$\{q_0\}$
q_2	ϕ	$\{q_3\}$
q_3	$\{q_0\}$	ϕ
$*q_4$	ϕ	ϕ

Extended Transition function for NFA

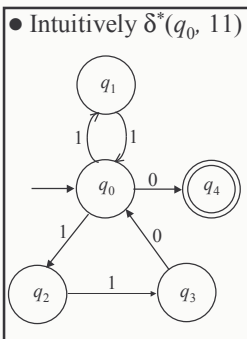
- Let $M = (Q, \Sigma, q_0, A, \delta)$ be a NFA.
- The function

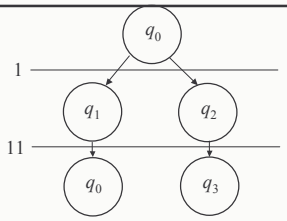
  $\delta^*: Q \times \Sigma^* \rightarrow 2^Q$

  is defined as follows:
  - For any $q \in Q$, $\delta^*(q, \Lambda) = \{q\}$
  - For any $q \in Q$, $y \in \Sigma^*$ and $a \in \Sigma$:

$$\delta^*(q, ya) = \bigcup_{r \in \delta^*(q,y)} \delta(r, a)$$

- Intuitively $\delta^*(q_0, 11)$ is:





$\delta^*(q_0, 11) = \{q_0, q_3\}$

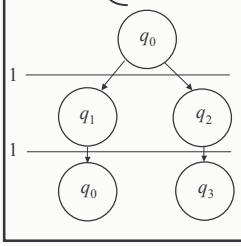
● Formally, $\delta^*(q_0, 11)$ is:

$\delta^*(q_0, 11)$ has the form $\delta^*(q_0, ya)$

Def. of δ^*:

$$\delta^*(q, \Lambda) = \{q\}$$

$$\delta^*(q, ya) = \bigcup_{r \in \delta^*(q,y)} \delta(r, a)$$



$r \in \delta^*(q_0, \Lambda) = \{q_0\}$

$\delta^*(q_0, \Lambda 1) = \delta(q_0, 1) = \{q_1, q_2\}$

$r \in \delta^*(q_0, \Lambda 1) = \{q_1, q_2\}$

$\delta^*(q_0, \Lambda 11) = \delta(q_1, 1) \cup \delta(q_2, 1)$

$= \{q_0, q_3\}$

Acceptance by an NFA

● Let $M = (Q, \Sigma, q_0, A, \delta)$ be a NFA.

– The string $x \in \Sigma^*$ is accepted by M if

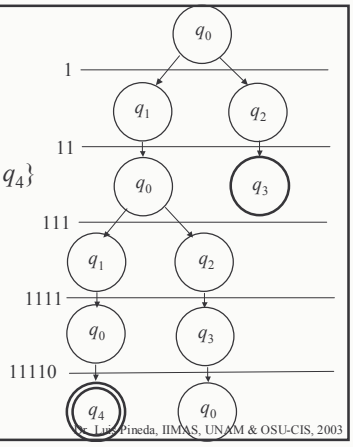
$\delta^*(q_0, x) \cap A \neq \emptyset$

– The language recognized by M is the set $L(M)$ of all strings accepted by M

– For any language $L \subseteq \Sigma^*$, L is recognized by M if $L = \mathbf{L}(M)$

● Accepting 11110:

$\delta^*(q_0, \Lambda 11110) = \{q_0, q_4\}$

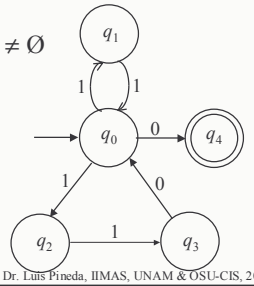


● Accepting 11110:

$\delta^*(q_0, \Lambda 11110) = \{q_0, q_4\}$

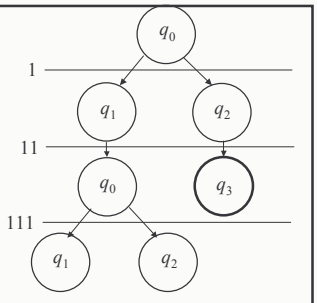
$A = \{q_4\}$

$\delta^*(q_0, \Lambda 11110) \cap A = \{q_4\} \neq \emptyset$



● But if the string is 111:

$\delta^*(q_0, \Lambda 111) = \{q_1, q_2\}$



● Accepting 111:

$\delta^*(q_0, \Lambda 111) = \{q_1, q_2\}$

$A = \{q_4\}$

$\delta^*(q_0, \Lambda 111) \cap A = \emptyset$