Work in Diagrammatic reasoning in AI and Cognitive Science

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Dr. Pineda's work started since his doctoral dissertation in the Centre for Cognitive Science at the University of Edinburgh, where he developed a study about the semantics of diagrams for the definition and interpretation of graphical or visual languages, with applications to human-computer interaction. He used a logical framework for representing diagrams and discovered an interesting kind of intensional properties of logical representations for geometric and diagrammatic objects. Extensional representations obey Leibniz' Law, according to which the substitution of equals for equals does not change the referent or truth value of the expression in which the substitution is made; however, as Frege clearly argued in his seminal work on sense and denotation, this property does not hold for natural language expressions in the so-called opaque contexts. For instance, although "the Morning star" and "the Evening star" are both names of Venus, the sentence "Pete thinks that the Morning star is Venus" does not entail "Pete thinks that the Evening star is Venus" because Pete might not know that Venus has this latter name too. In his earliest papers and his dissertation Dr. Pineda showed that an analogous situations occurs for diagrammatic descriptions. His reasoning goes as follows: suppose that there is a diagram in which three lines *A*, *B* and *C* intersect in a common position; then the expression "the position of the intersection between *A* and *B* is the same as the position of the intersection between *B* and *C*" is true. It also follows that the intersection between *A* and *C* is located at the same position. However, if the line *C* is dragged down, say by an interactive operation upon the line on a computer display, the equality becomes a false statement and the implication does not longer follow, despite that the lines in questions are the same. Although diagrammatic objects can have several alternative descriptions in a given diagrammatic state, like the position of the intersection in example above, and substitution of equals by equals is allowed in that state, if the diagram is changed the diagrammatic descriptions remain the same but their semantic values may change, so Leibniz's law does not hold. In this sense, the geometric space can be thought of as large opaque context in which diagrammatic representation and reasoning takes place. In Pineda's view, in the same way that substitution is not allowed in opaque contexts because the knowledge and belief states of agents differ, diagrammatic expressions and descriptions vary their extensional values simply because the things in the world move, although the identity of the objects is preserved; in the limiting case, a dot on a piece of paper has only two properties, its position and color, however, if the dot is moved to a different position and its color is changed, it is still the same dot, despite that its only two properties are contingent and have different values in different diagrammatic states. These observations led Dr. Pineda to suggest that symbols on diagrammatic representation should be thought of as "rigid designators" along the lines suggested by Kripke for natural language proper names. On the other hand, descriptions may encode complex constraints that hold for a large number of states, and some times for all possible states, so intensional descriptions can be used the represent diagrams and diagrammatic sequences very effectively. This discovery led to a form of object-oriented graphics, which had the advantage that the logical framework allowed to reason about the diagrams in quite a simple and natural way.

Equipped with this theoretical machinery, Dr. Pineda developed a system called Graflog for intelligent drafting. In its original version it was possible to express diagrammatic configurations and state their interpretation in relation to an arbitrary domain through a multimodal graphics and natural language interactive interface, and ask questions about the meaning of the diagrams and also about the consequences of the facts stated diagrammatically. This work was published in the paper "Understanding Drawings Through Natural Language" in the journal *Computer Graphics Forum* in 1988. The system could also solve problems that required heterogeneous reasoning, where information was expressed through diagrams and natural language or logic, but neither of the modalities had complete information, and problem solving required the use of information expressed in both of the modalities in a coherent and systematic way. This was published in the paper "Semantics and Graphical Information" in conjunction with his PhD Supervisor, Dr. Ewan Klein, in 1990, and it is a recurrent topic in his subsequent work. Then he also showed that some constraint satisfaction problems that had been solved through linear methods or through other constraint satisfaction techniques, like relaxation, could be solved using intensional descriptions directly by simply interpreting the expressions (i.e. producing their extensional value) in the relevant diagrammatic states. This result was published in the paper "Reference, Synthesis and Constraint Satisfaction", also in *Computer Graphics Forum* in 1992. A later version of the Graflog system was able to produce technical diagrams, subject to an arbitrary set of geometric constraints, through a reasoning process based on highly structured drafting rules. The system was non-deterministic and could find and display several solutions for arbitrary problems, some of which were unexpected and even surprising for human interpreters. This work was described in several conference proceedings and was summarized in the paper "On Computational Models of Drafting and Design" published in the journal *Design Studies* in 1993.

At his return to Mexico, Dr. Pineda continued elaborating his theory about the semantics of graphics with applications to problem solving in CAD systems, and in 1998 published the paper "Synthesis of Solid Models of Polyhedra from their Orthogonal Views using Logical Representations" in *Expert Systems with Applications*, in conjunction with one of his students. In this paper they presented a fully declarative representation with a well-defined semantics of 2-D and 3-D geometric objects, and also of the rules and problem solving process for modeling this kind of reasoning. Then, he continued working on the relation between diagrammatic and natural language interpretation, and this work achieved a full formalization in the paper "A Model for Multimodal Reference Resolution", which was published in the journal *Computational Linguistics* in 2000. In this paper he investigated further the notion that diagrams can be represented through logical languages and translated into natural language expressions directly, following very closely the spirit and notational framework provided by Montague in his semantic and semiotic program. This paper also presents an algorithm to interpret multimodal diagrammatic and textual presentations, like diagrams with a caption, which permits to label graphical symbols and also to determine the referent of indexical linguistic terms, like demonstratives and indexical pronouns, on the basis of a constraints expressed in the both of modalities involved.

At that time, Dr. Pineda focused more directly on problems related to the expressive power of diagrams, and discussed whether these are limited to express concrete information, or whether abstractions can be expressed through diagrams and used effectively in reasoning and problem solving. He also addressed problems related to the specification and interpretations of notations, and pointed out that the reinterpretation of the symbols and configurations under different notations is an essential aspect of problem-solving, learning and discovery. These reflections were published in the article "Diagrammatic Inference and Graphical Proof", published as a book chapter in the *Applied Logic Series* in 2000, where an abstraction hierarchy for diagrams supporting logical reasoning, and also for diagrammatic geometric and arithmetic proofs is presented. Contrary to the widespread belief that diagrams are limited in the abstractions that they can express, in this article it is claimed that diagrams representing geometric and arithmetic proofs do express unlimited abstractions under the appropriate interpretation conditions. Later on Dr. Pineda addressed directly how diagrammatic proofs can be generated and represented computationally; he refined and enriched the machinery that had been developed along the years and incorporated the new intuitions about the expressive power of diagrams; with the resulting theoretical framework and programming environment he developed and tested a theory for the synthesis of the concepts expressed through diagrams in diagrammatic proofs. This theory was presented in the paper "Conservation principles and action schemes in the synthesis of geometric concepts" that was published in the journal *Artificial Intelligence* in 2007. In this paper it is shown that valid diagrammatic reasoning requires the use of general concepts of equality, which are formalized as higher-order functions and, after Piaget's corresponding notion, named "conservation principles". He also introduced highly structured synthetic machinery for synthesizing the diagrams appearing in diagrammatic theorems and proves, which he called "action schemes". Dr. Pineda also developed a description machinery to be able to refer to diagrammatic objects and configurations that emerge in the context of other symbols in a fully abstract way. For this he introduced a new logical operator, which he called "geometrical descriptor", that resembles Hilbert's ε-operator, which permits to define terms that denote arbitrary set of objects that satisfy certain condition, but also has a functional interpretation and can be evaluated for specific arguments, and resembles in this respect Russell's ι-operator used to build terms that denote a unique object satisfying certain property. With these theoretical devises he introduced the notion of diagrammatic derivation, and was able to produce a semi-automatic diagrammatic proof of the theorem of Pythagoras by the first time in the AI literature. Given the AI interest in theorem proving, the lack of a theorem-proving system able to carry such a kind of fundamental proofs posed interesting philosophical and methodological questions, which are also addressed in this paper. In Dr. Pineda's theory geometric and arithmetic concepts are represented through functions in the lambda calculus. The concept of the theorem of Pythagoras, for instance, is represented through a function of four geometric arguments, a right-triangle and three squares, and the value of the function is true for a given set of objects if they stand in the Pythagorean relation and false otherwise. The program permits also to evaluate the functions that it itself generates, and computes the extension of the synthesized concepts for arbitrary sets of arguments of the proper types. These functions are not expressed in advance in the representational structures of the system, and the program can be also thought of as a learning or discovery program. This machinery was also used to generate inductive diagrammatic proofs, like the proof of the sum of the odd numbers, in a simple and general way. In its general discussion, and contrary to theorem-proving traditions that have a strong syntactic orientation, Dr. Pineda supports that diagrammatic reasoning is essentially a semantic process. This paper has in addition at least three important contributions: 1) refutes the theory of graphical specificity, advanced by Stenning and Oberlander, which holds that there is a limit to the abstractions that can be expressed through graphical representations, b) reviews and refines a classification of diagrammatic proofs suggested by Jamnik and adopted by others, and shows that the underlying description and interpretation processes are the same for both standard and inductive diagrammatic proofs, as the concept of mathematical induction is a full abstraction that can be better expressed through language, and c) questions the widely accepted so-called knowledge representation trade-off introduced by Brachman and Lavesque; the claim is that there is a compromise between the expressive power of a representational language and its computational tractability so, on the one hand, reasoning with concrete information is simple but abstraction cannot be expressed but, on the other, the expression of abstractions makes reasoning impractical. In this regard, Dr. Pineda argues that the suggested trade-off depends on the underlying assumption that reasoning is essentially a syntactic guided process; however, if a semantic framework is adopted instead, reasoning with abstractions has a much more reduced computational cost. Concrete problems can be expressed in languages with low expressive power, like propositional logic or regular expressions, if the problem domain is finite and not too large; however, if the problem demands abstraction a highly expressive language, like first order or a higher-order language is required, but reasoning needs to have a strong semantic orientation. He also argues that what is hard is to produce the right abstractions, a process that requires the interaction of perception and thought, and perhaps schematic and abductive inference strategies, but when the right abstractions are in place, the solution of complex problems can be found quite directly. This is illustrated with the theory of conservation principles and actions schemes, and it is also very consistent with common sense intuition. Finally, in this paper, Dr. Pineda also claims that an abstraction is a small representational object, that can be stored and manipulated in memory and thought processes as a single unit, but that its interpretation has a very large, possible infinite extension, and can be produced with a very reduced computational power.