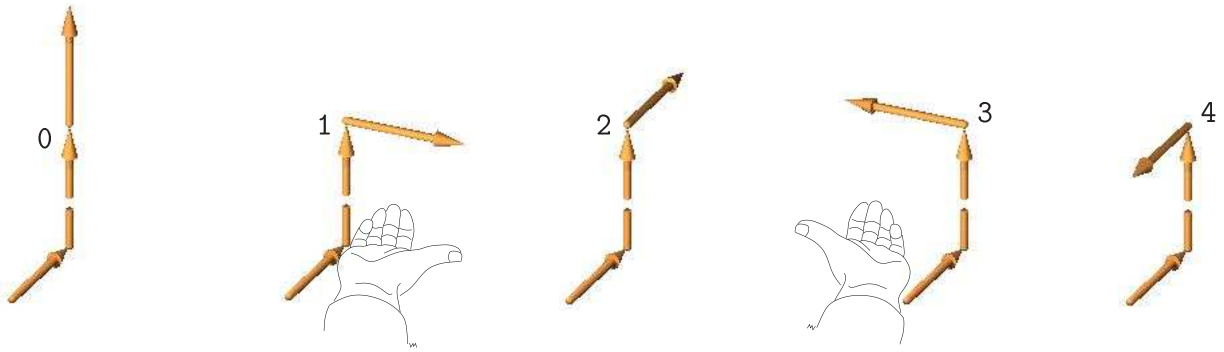
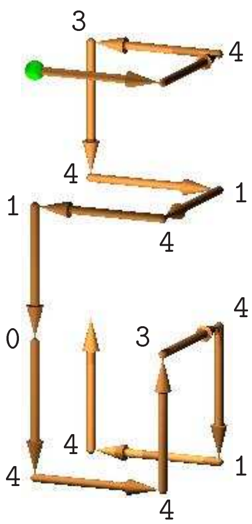
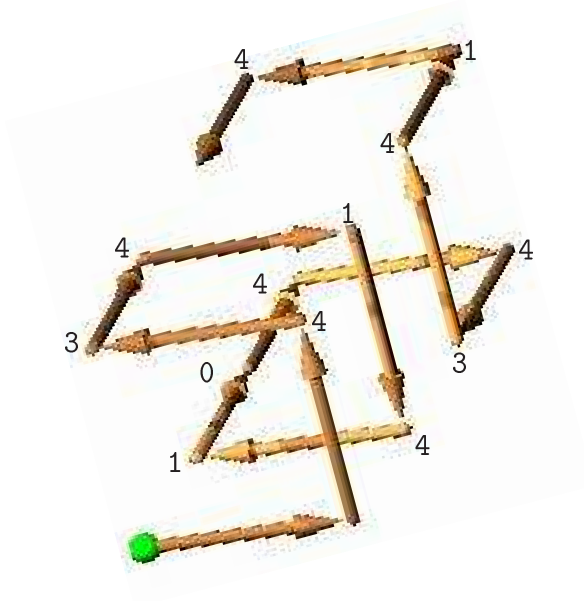
Código cadena





4341410443414

Cadena

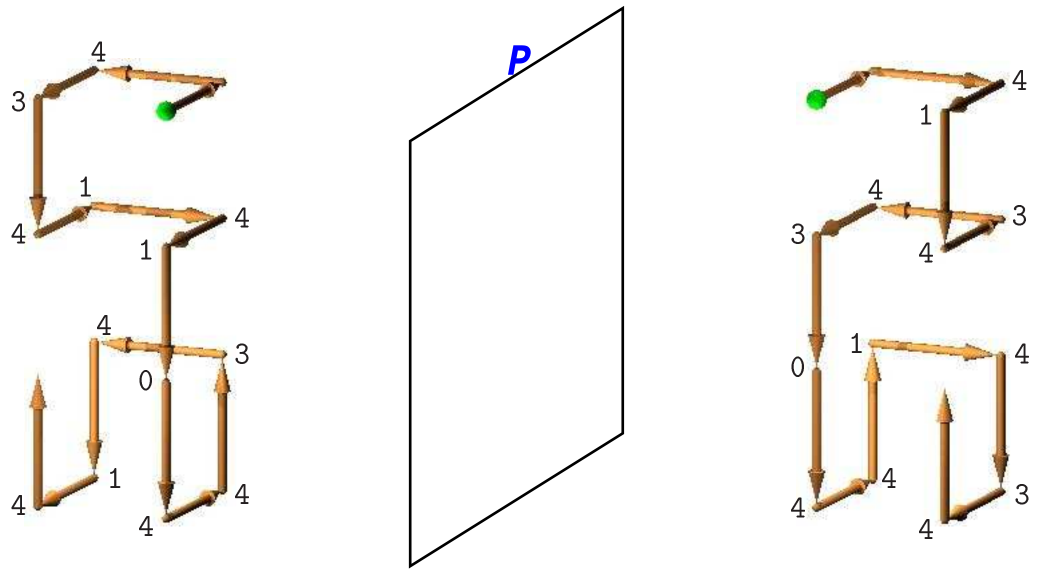


El código es invariante ante rotaciones, traslaciones y expansiones.

$$label(\boldsymbol{a},\boldsymbol{b},\boldsymbol{c}) = \begin{cases} 0, & \text{si } \boldsymbol{c} = \boldsymbol{b}; \\ 1, & \text{si } \boldsymbol{c} = \boldsymbol{a} \times \boldsymbol{b}; \\ 2, & \text{si } \boldsymbol{c} = \boldsymbol{a}; \\ 3, & \text{si } \boldsymbol{c} = -(\boldsymbol{a} \times \boldsymbol{b}); \\ 4, & \text{si } \boldsymbol{c} = -\boldsymbol{a}; \end{cases} \qquad (\ |\boldsymbol{a}| = |\boldsymbol{b}| = |\boldsymbol{c}| = 1,\ \boldsymbol{a} \perp \boldsymbol{b}\ ).$$

Operaciones algebraicas

Reflexión en un plano P

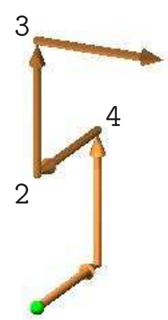


4341410443414

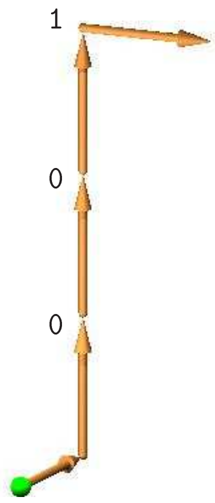
$$\mathit{mirror}\ \mathtt{4341410443414} = \mathtt{4143430441434}$$

$$\mathit{mirror}\ e_1 e_2 \cdots e_n = \widehat{e_1}\widehat{e_2}\cdots\widehat{e_n}, \quad \text{con} \quad \hat{e} = \begin{cases} 3, & \text{si } e = 1; \\ 1, & \text{si } e = 3; \\ e, & \text{si } e = 0, 2 \text{ ó } 4. \end{cases}$$

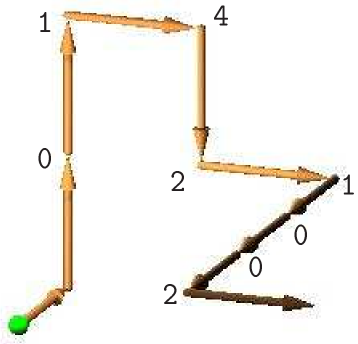
Inversión



$$\mathit{inv}\ \mathtt{423} = \mathit{reverse}\ 423 = \mathtt{324}$$



$$\mathit{inv}\ \mathtt{001} = \mathtt{001}$$



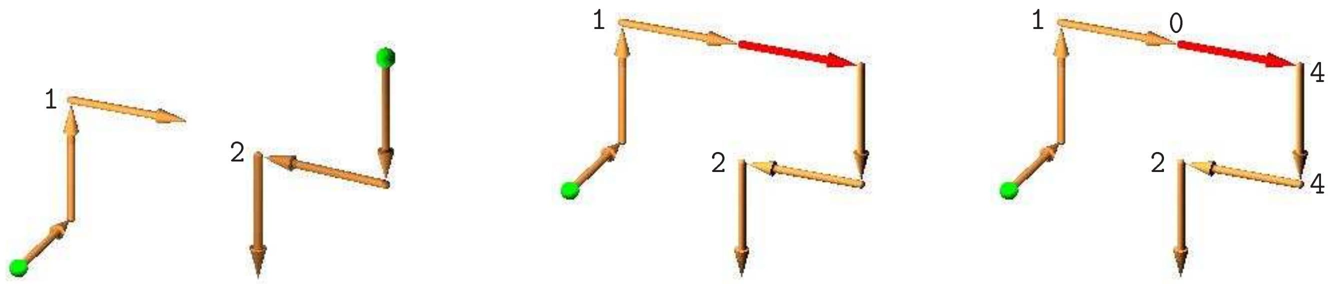
$$\begin{aligned}&\mathit{inv}\ \mathtt{01421002}\\ &= \mathit{inv}\ (\mathtt{01} +\!\!+ \mathtt{421} +\!\!+ \mathtt{002})\\ &= \mathit{inv}\ \mathtt{002} +\!\!+ \mathit{inv}\ \mathtt{421} +\!\!+ \mathit{inv}\ \mathtt{01}\\ &= \mathtt{002} +\!\!+ \mathtt{124} +\!\!+ \mathtt{01} = \mathtt{00212401}\end{aligned}$$

Sea $c = e_1 e_2 \cdots e_n$, $e_n \neq 0$,

$$\mathit{inv}\ c = \begin{cases} \mathit{reverse}\ c, & \text{si } e_1, e_2, \ldots, e_n \neq 0;\\ c, & \text{si } e_1, e_2, \ldots, e_{n-1} = 0, e_n \neq 0;\\ \mathit{inv}\ c_m +\!\!+ \cdots +\!\!+ \mathit{inv}\ c_1, & \text{si } c = c_1 +\!\!+ \cdots +\!\!+ c_m \text{ (con } c_1, \cdots, c_m\\ & \qquad \text{como es requerido en caso 1 ó 2).} \end{cases}$$

Ligado

$l = 044$



$a = 1$ $b = 2$

$liga\ a\ l\ b$
$= 1 \mathbin{+\!\!+} 004 \mathbin{+\!\!+} 2 = 10442$

Una ligadura l ***es una cadena invertible de longitud mayor que 1.***

$$liga\ a\ l\ b = a \mathbin{+\!\!+} l \mathbin{+\!\!+} b.$$

Propiedades

$$mirror\ (mirror\ c) = c$$

$$mirror\ (c_1 \mathbin{+\!\!+} \cdots \mathbin{+\!\!+} c_n) = mirror\ c_1 \mathbin{+\!\!+} \cdots \mathbin{+\!\!+} mirror\ c_n$$

Si `1` ***y*** `3` ***no ocurren en*** c***, entonces***

$$mirror\ c = c$$

Si $c, c_1, \ldots, c_n$ ***son invertibles, entonces***

$$inv\ (mirror\ c) = mirror\ (inv\ c)$$

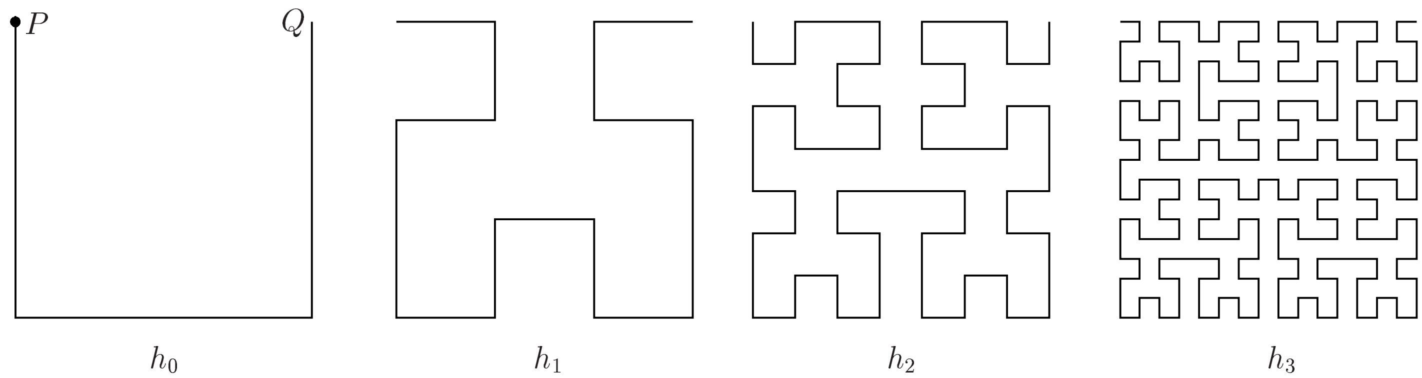
$$inv\ (inv\ c) = c$$

e

$$inv\ (c_1 \mathbin{+\!\!+} \cdots \mathbin{+\!\!+} c_n) = inv\ c_n \mathbin{+\!\!+} \cdots \mathbin{+\!\!+} inv\ c_1$$

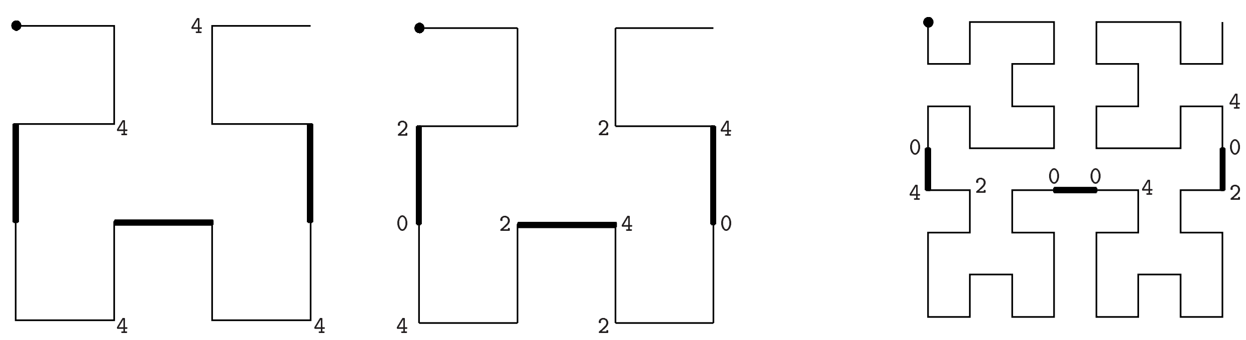
Una aplicación: Construcción de sucesiones de Hilbert

En 1890 Giuseppe Peano ideó curvas continuas que pasan por todos los puntos de un cuadrado. En 1891 David Hilbert definió una sucesión de curvas continuas, uniformemente convergente a una curva de Peano. Las primeras curvas de esta sucesión son similares a las siguientes:

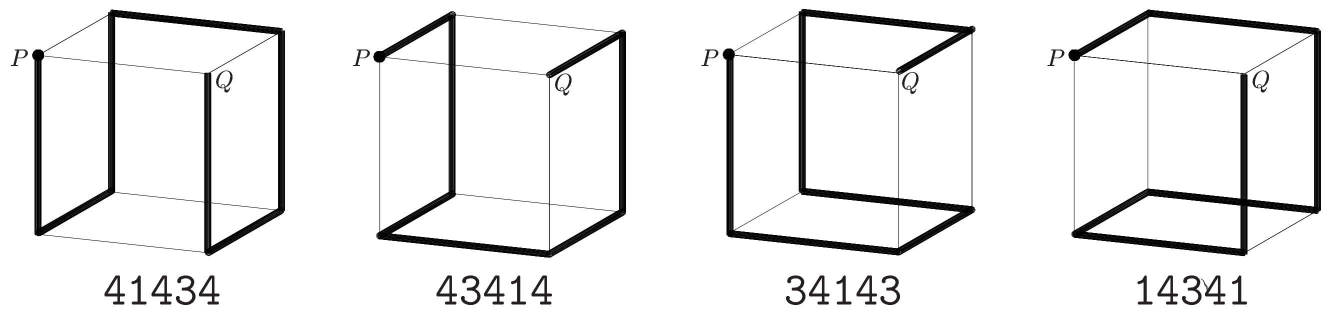



```
h 0     = "4"
h (n+1) = if even n then
            hn ++ "204" ++ hn ++ "242" ++ hn ++ "042" ++ hn
          else
            hn ++ "042" ++ hn ++ "004" ++ hn ++ "204" ++ hn
          where hn = h n
```

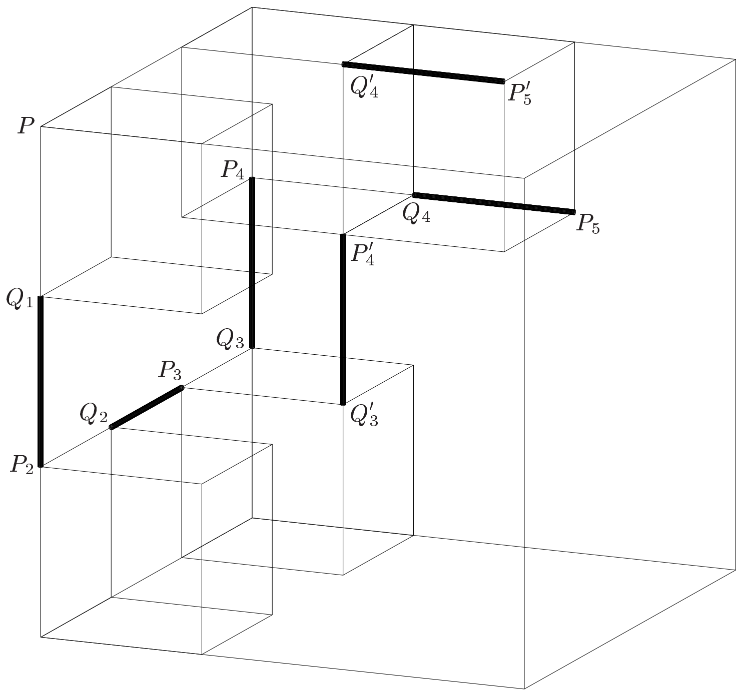
Definicion ejecutable, escrita en `Haskell`***, de una función recursiva*** `h` ***tal que*** `h n` ***resulta igual al código cadena de*** h_n ***(***$n \in \mathbb{N}$***).***



Sucesiones de Hilbert en $\mathbb{R}^3$



Posibilidades para h_0.



Posibilidades para h_{n+1}.

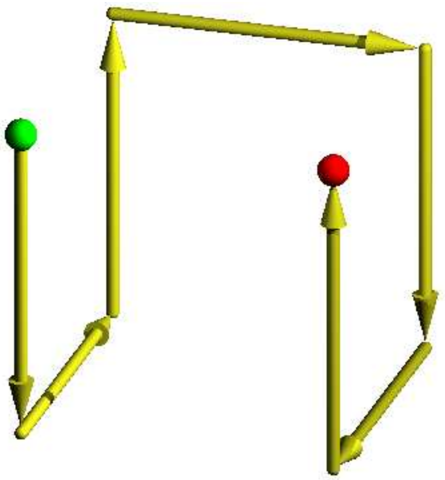
Pidiendo que h_n refleje sobre si misma en un plano $\perp$ a PQ, se debe cumplir

$$h_n = inv\,(mirror\,h_n),$$

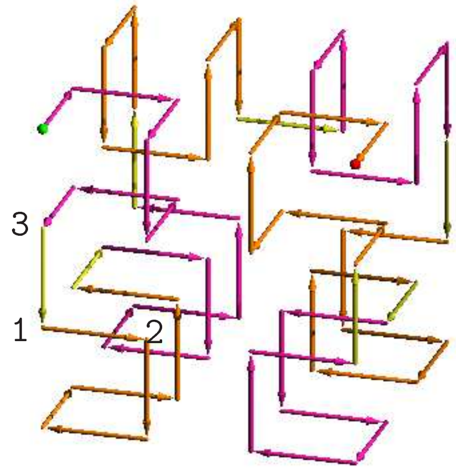
de donde se sigue que

$$h_{n+1} = h'_{n+1} +\!\!+\; l_{n+1} +\!\!+\; inv\,(mirror\,h'_{n+1}),$$

siendo h'_{n+1} el código de la primera mitad de h_{n+1} y l_{n+1} la ligadura entre las dos partes.



h_0

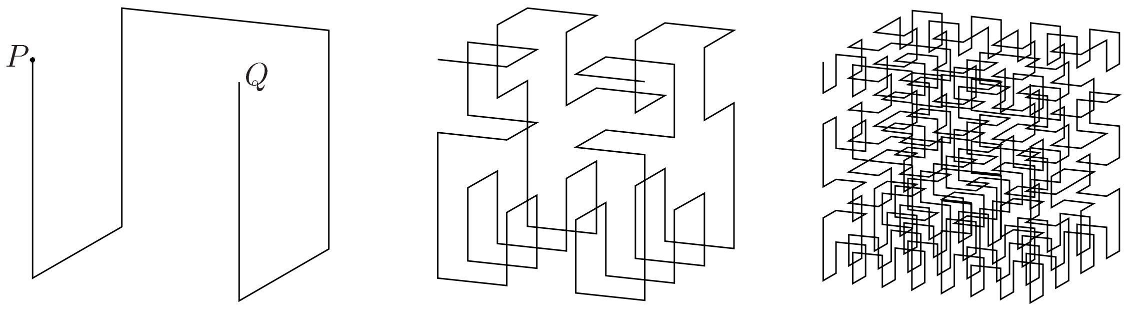


h_1

$$
\begin{aligned}
h_0 &= \texttt{41434} \\
h_1 &= (h_0 +\!\!+\, \texttt{312} +\!\!+\, h_0 +\!\!+\, \texttt{341} +\!\!+\, h_0 +\!\!+\, \texttt{203} +\!\!+\, h_0) +\!\!+\, \texttt{341} +\!\!+ \\
&\quad\ \mathit{inv}\,(\mathit{mirror}\,(h_0 +\!\!+\, \texttt{312} +\!\!+\, h_0 +\!\!+\, \texttt{341} +\!\!+\, h_0 +\!\!+\, \texttt{203} +\!\!+\, h_0)) \\
&= h_0 +\!\!+\, \texttt{312} +\!\!+\, h_0 +\!\!+\, \texttt{341} +\!\!+\, h_0 +\!\!+\, \texttt{203} +\!\!+\, h_0 +\!\!+\, \texttt{341} +\!\!+ \\
&\quad\ h0 +\!\!+\, \texttt{012} +\!\!+\, h0 +\!\!+\, \texttt{341} +\!\!+\, h0 +\!\!+\, \texttt{231} +\!\!+\, h0
\end{aligned}
$$

```
h 0     = "41434"
h (n+1) =
 case n `mod` 3 of                                                          0 ->
   hn ++ "312" ++ hn ++ "341" ++ hn ++ "203" ++ hn ++ "341" ++
   hn ++ "012" ++ hn ++ "341" ++ hn ++ "231" ++ hn
  1 ->
   hn ++ "203" ++ hn ++ "242" ++ hn ++ "041" ++ hn ++ "242" ++
   hn ++ "304" ++ hn ++ "242" ++ hn ++ "012" ++ hn
  2 ->
   hn ++ "041" ++ hn ++ "004" ++ hn ++ "312" ++ hn ++ "004" ++
   hn ++ "231" ++ hn ++ "004" ++ hn ++ "304" ++ hn
 where hn = h n
```

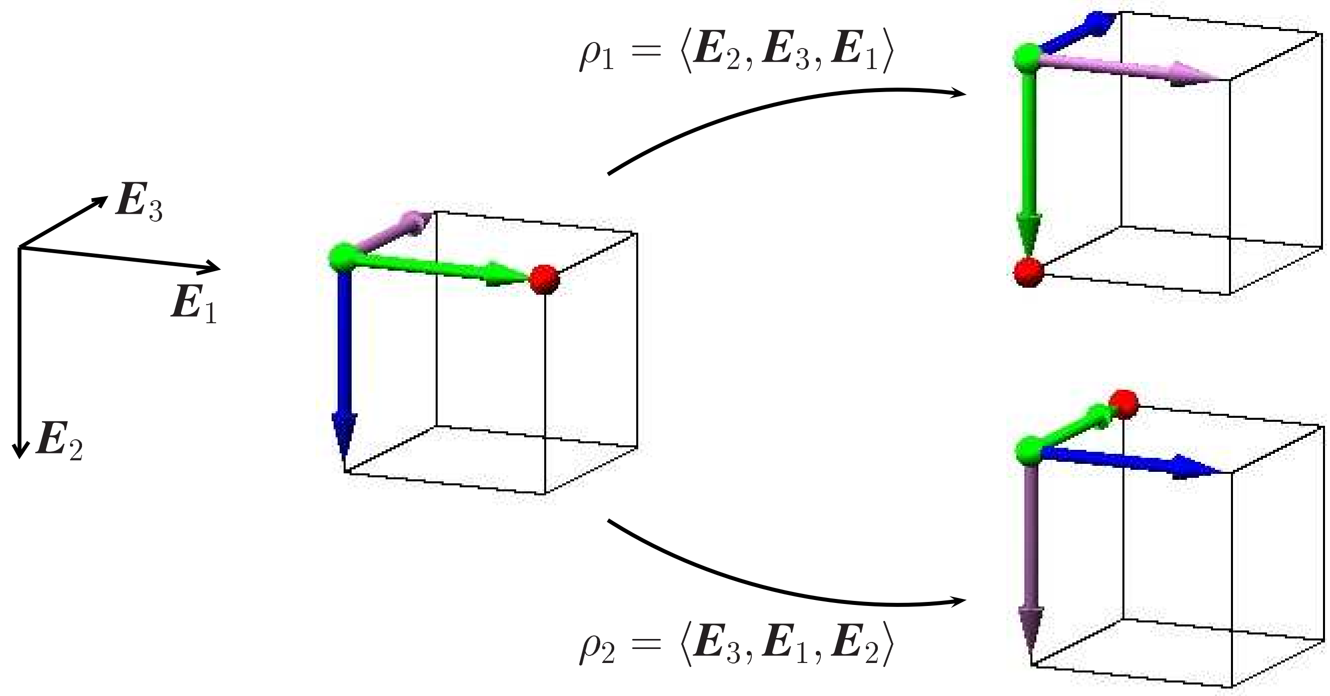
Definiendo h_{n+1} con copias de $mirror\, h_n$



. . .

```
h 0     = "41434"
h (n+1) =
 if even n then
  mhn ++ "104" ++ mhn ++ "143" ++ mhn ++ "032" ++ mhn ++ "143" ++
  mhn ++ "201" ++ mhn ++ "143" ++ mhn ++ "043" ++ mhn
 else
  mhn ++ "041" ++ mhn ++ "004" ++ mhn ++ "312" ++ mhn ++ "004" ++
  mhn ++ "231" ++ mhn ++ "004" ++ mhn ++ "304" ++ mhn
 where mhn = mirror (h n)
```

Computación de las ligaduras



Las 48 simetrías del cubo corresponden a:

$$\boldsymbol{E}_1 \mapsto \pm \boldsymbol{E}_i,\ \boldsymbol{E}_2 \mapsto \pm \boldsymbol{E}_j,\ \boldsymbol{E}_3 \mapsto \pm \boldsymbol{E}_k, \quad \mathrm{con}\{i,j,k\} = \{1,2,3\}$$

y se pueden representar en la forma $\langle \pm \boldsymbol{E}_i, \pm \boldsymbol{E}_j, \pm \boldsymbol{E}_k \rangle$.

De los dos primeros lados h_n *se obtienen los últimos dos mediante*

$$\sigma = \langle \boldsymbol{E}_1, -\boldsymbol{E}_2, -\boldsymbol{E}_3 \rangle.$$

Como h_0 *inicia con* $\boldsymbol{E}_2$ *y* $\boldsymbol{E}_3$*, los lados que determinan la primera ligadura de* h_1 *los da la lista*

$$[\,\rho_1\,\sigma\,\boldsymbol{E}_3,\ \rho_1\,\sigma\,\boldsymbol{E}_2,\ \boldsymbol{E}_2,\ \rho_2\,\boldsymbol{E}_2,\ \rho_2\,\boldsymbol{E}_3] = [-\boldsymbol{E}_1, -\boldsymbol{E}_3, \boldsymbol{E}_2, \boldsymbol{E}_1, \boldsymbol{E}_2].$$

Pero

$$label(-\boldsymbol{E}_1, -\boldsymbol{E}_3, \boldsymbol{E}_2) = 3$$
$$label(-\boldsymbol{E}_3, \boldsymbol{E}_2, \boldsymbol{E}_1) = 1$$
$$label(\boldsymbol{E}_2, \boldsymbol{E}_1, \boldsymbol{E}_2) = 2$$

Por lo tanto, el código de la ligadura es 312.

Referencias

[1] E. Bribiesca, A chain code for representing 3D curves, *Pattern Recognition*, Vol. 33, No. 5, pp. 755-765 (2000).
(`http://www.elsevier.nl/locate/elecacc`).

[2] E. Bribiesca and C. Velarde, A Formal Language Approach for a 3D Curve Representation, to appear in *Computers Math. Applic.* journal, 2000.