A mathematical method to dilute contaminated cereal lots for animal feed

Magda Carvajal*

Departamento de Botánica

Instituto de Biología, UNAM

Apdo. 70-233, Coyoacán

04510 México D.F.

magdac@servidor.unam.mx

Carlos Velarde†

IIMAS, UNAM

Apdo. 20-726

01000 México D.F.

velarde@servidor.unam.mx

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*To whom correspondence should be addressed. Telephone number: (525)6225693 ext 242. Fax: (525)5501760.

†Partially done at: Unidad de Investigación en Cómputo Aplicado, DGSCA, UNAM.

Abstract

Countries around the world have their cereal grains contaminated. Given that chemical decontamination is very expensive, after stopping the fungal growth and homogenizing the maize, a common process used is the mixture ('dilution') of the contaminated cereal with a healthy one. This method diminishes the levels of toxins, some of them very carcinogenic. This 'dilution' is usually performed without any scientific background and positive results are rarely obtained. To illustrate our proposed solution, several examples involving different lots with one or more contaminants are shown mathematically. This work presents a mathematical method to 'dilute' contaminated cereal with healthy one obtaining useful grain for animal consumption, following legal requirements. A mathematical approach is usually feared by agricultural, storage and administrative workers. These workers must only learn to feed the correct information into a computer working with a software system like Mathematica.

1 Introduction

Tropical countries have thousands of tons (1 ton = 1000 kg) of cereals of different quality grades, with colorants urea, insect parts, ammonium or contaminants. Many times mycotoxin contaminated maize is used for feed. Cereals are unexpensive and are staple feed in all the countries. Decontaminating methods are expensive (Dichter, 1987), they alter cereal flavors, darken cereal color, and odor of ammonium stays, making decontaminated cereals not easily accepted for animal consumption. In some cases the need for food is so extreme that contaminated cereals are used. A common used method to decontaminate lots of contaminated cereals is to mix them with 'clean' cereal, a method called 'dilution'. Dilution allows for mixed cereal to feed adult male bovines.

Mycotoxins have heterogeneous distribution and this fact has to be solved before applying the present method. There are 3 problems that have to be solved as previous conditions to dilute cereal lots with different mycotoxin contamination: 1. Fungal growth: Mycotoxin producer fungi have to be eliminated by heat or paralized by propionic acid, if the fungus is reproducing it is impossible to use this mathematical technique, because the fungus will be producing new toxin without control. The fungus can be eliminated with heat and propionic acid prevents new contamination, making it static, and the mixture can be done; 2. Heterogeneous distribution of mycotoxins in a batch: Another condition is that cereal lots have to be homogenized, ground into powder or fluor and mixed thoroughly so mycotoxin contamination can be well distributed. Normally the grains have different quantity of mycotoxins and if we mix whole grains the lots will never be homogenized becaused mycotoxins grow in spots. So we need to ground the cereal, mix the batches and later do the pellets for animal feed; and 3. Sampling: Some errors associated with mycotoxin tests are sampling, subsampling and analytical mistakes that affect the accuracy of quantitative mycotoxin analysis (Dickens and Whitaker, 1982; Dickens and Whitaker, 1986). Surveillance and quality control procedures involve sampling, sample preparation and analysis, reliable data results depend on the effectiveness of these steps. The sampling has to be performed as accurately as possible and the sample collected has to be representative of the batch of feed investigated. The collection of a biased sample invalidates the resultant analytical data. Studies already performed indicate that representative samples of commodities composed of large particles (corn and oilseed kernels) should be 10 kg in weight at least and composed of approximately 100 incremental samples, and samples of oilseed cakes and meal, should be composed of 50 incremental samples which afford a composite sample of approximately 5 kg in weight (Coker et al., 1995).

The problem is, what to do with the contaminated part of the crop? To prevent its consumption, sometimes the contaminated lots are stained, but the grain is never destroyed. It is frequently mixed with healthy maize to make it edible for animals, but the mixture is done in a domestic way, without mathematical support or quality control, and the problem is not satisfactorily solved.

Although the governments of the different countries have legislations to prevent the risk of ingestion of contaminated cereals, the truth is that aflatoxins and mycotoxins are frequently present in cereals and in industrialized food (Egmond, 1989). The maximum accepted level in U.S.A. and Mexico (Jelinek, 1987) is 20 mg/ton (= 20 μg/kg) for total aflatoxins in poultry feed. Analyzed samples usually show higher levels of aflatoxins than those allowed. In other countries with different legal limits, adjustments have to be made.

This paper intends to provide an accurate mathematical method for mixing mycotoxin contaminated cereal, with a clean cereal lot, in order to obtain a lot of grain with less than 20 μg/kg with previous homogenization and stopping the fungal growth. Four examples are provided. In the first example, a cereal lot with a contaminant is diluted in a clean one. The second example shows how to mix two lots with two different mycotoxin levels. A third example shows three lots with two contaminants and, in the fourth example, a mathematical solution for n lots with m contaminants is presented.

2 Mathematical Methodology

Given two lots of cereal with different levels of contaminant concentration, the goal is to obtain the largest amount of mixture with an accepted level of contaminant concentration (Bhat, 1987). This work presents a way of solving the more general problem of how to obtain the biggest amount of mixture composed by parts taken from n lots of cereal with m contaminats, in such a way that in the final mixture the concentration of each contaminant does not exceed a corresponding given limit.

Our problem is recognized as a classical one in "Linear Programming" (Gass, 1964; Hadley, 1962; Smythe and Johnson, 1966), a mathematical branch which deals with the optimisation of linear functions subject to linear constraints. There are well known methods —as the *simplex* method and Karmarkar's method (Ralston, 1993; Strang, 1986; Zionts and Ramesh, 1993)— to solve similar problems. We could follow one of those methods to solve our mixture problems, but since now there are commercial software packages to carry out the cumbersome computations involved, we prefer instead to present a solution based on one of such packages. We only need to take care of the algebraic formulation of the problem, as the modern computer packages are able to receive those symbolic formulas as data (Abell and Braselton, 1992; Wolfram, 1988).

Before presenting the solution to the general problem of n lots with m contaminants, to get some insight we present three particular cases. We begin with the simple case of two lots with one contaminant, we then consider two lots with two contaminants, and finally we treat three lots with two contaminats. We chose these cases because they accept natural geometric interpretations, and we believe geometry is a good vehicle for the intuition.

The first problem is as follows: Given two lots of cereal with different levels of contaminant concentration, the goal is to obtain the largest amount of mixture within a legal limit of the contaminant.

In this optimisation problem, the purpose is to maximize the amount of mixture, subject to one constraint that is the concentration of contaminant permissible for animal feed in the final mixture.

To formulate the problem in mathematical terms the following symbology is used. Let A be the quantity in tons of the given healthy cereal lot with a contaminant concentration of a mg/ton, let B be the quantity in tons of contaminated cereal with a contaminant concentration of b mg/ton, and let k mg/ton be the contaminant legal limit accepted for the final mixture.

We want to determine the maximum quantity of mixture obtained by mixing x tons of healthy maize with y tons of contaminated maize, and whose contaminant concentration does not exceed k. Thus, we want to know the values of x and y that maximize the mixture quantity:

$$x + y \tag{1}$$

subject to the given restriction in contaminant concentration. The $x + y$ tons of mixture carry $ax + by$ mg of contaminant, and this quantity must not exceed the limit of $k(x+y)$ mg of contaminant. This can be written shortly in mathematical language:

$$ax + by \leq k(x + y)$$

By simple algebraic manipulations, this inequality can be rewritten as:

$$(k - a)x + (k - b)y \geq 0 \tag{2}$$

Once the problem has been translated into the form shown in the formulas (1) and (2), it can be immediately recognized as a classical problem in Linear Programming, and can be solved using the techniques developed on this subject. But being our example so simple in nature, to get some insight we prefer first to present a solution in terms of elementary Algebra and Analytical Geometry. Later, in Example 2 we will show a way for solving the problem with the aid of software for linear programming.

As we know that all the lot of clean maize must be used, this means that $x = A$. So, it only remains to determine the value for y. Substituting A for x in (2) and then solving for y we obtain:

$$y \leq \frac{k-a}{b-k}A$$

Thus, the maximum value for y that can be used is given by the right hand side of this inequality, that is:

$$\frac{k-a}{b-k}A \tag{3}$$

Then, if the weight of the contaminant contaminated lot does not exceed this value, all the B tons can be used to form the mix. Otherwise, precisely the quantity (3) taken out from the B tons must be used.

Example 1 Suppose $A = 80$ tons, $B = 60$ tons, $a = 15$ mg/ton, $b = 100$ mg/ton, $k = 20$ mg/ton. Then, substituting appropriately these data in (3) we calculate the maximum quantity of contaminated maize that can be used:

$$\frac{k-a}{b-k}A = \frac{20-15}{100-20}80 = 5 \text{ tons}$$

So, from the 60 tons (B contaminated lot) we can only use 5 tons of the contaminated lot, to be diluted in the 80 tons of clean maize. In this way we obtain 85 tons of mixture, with a contaminant concentration equal to the legal limit of 20 mg/ton. □

A geometrical interpretation over the Cartesian plane is presented in Figure 1. Each point (x, y) of the rectangle represents a possible mixture, obtained by mixing x tons of healthy maize with y tons of contaminated one. From these possible mixtures, those which satisfy the condition (2) correspond to points in the shaded region. The points on the slanted line represent mixtures with a concentration equal to the fixed k limit.

These are the points for which the equality in (2) holds. The vector $(k-a, k-b)$ —which is formed by the coefficients in the left-hand side of (2)— is graphically represented by an arrow perpendicular to the slanted line, and its direction signals the half-plane whose points satisfy the inequality in (2). The segment $\overline{AP}$ represents acceptable mixtures for which all the healthy lot is used. In particular, point P corresponds to the mixture for which the highest amount of contaminated maize is used. Note that in Figure 1–(i) P corresponds to a mixture formed with the totality of both the contaminated and healthy lots. This mixture will have a contaminant concentration lower than k, given by the formula $(aA+bB)/(A+B)$. On the other hand, in Figure 1–(ii), P stands for a mixture where only a part of the contaminated lot can be mixed with the clean one —as illustrated in Example 1. This amount, as we saw, is given by (3). In this case, the mixture will have a contaminant concentration equal to k.

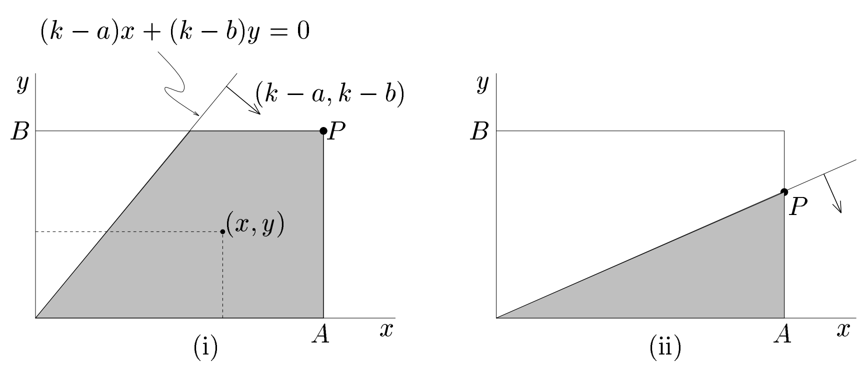

Figure 1: *Point P corresponds to the maximum quantity of mixture.*

Example 2 Here we show how the problem of Example 1 can be solved with the aid of a software system like Mathematica (Abell and Braselton, 1992; Wolfram, 1988).

Once we are in an interactive session, we give as input the following lines:

```
ConstrainedMax[  x+y,
                 { (20-15)x + (20-100)y >= 0,
                   x <= 80,  y <= 60 },
                 { x, y } ]
```

where ConstrainedMax is a three-argument built-in operator for solving linear programming problems of the kind we are considering here. The first argument is a transcription of (1), the linear function to be maximized. The second argument, written between braces in the second and third lines, comprises the linear constraints; the first inequality is no other than (2), but in it substituted the corresponding given values for k, a, and b, and with the symbol "$\geq$" presented in the form ">=", which is a convenient way for writing "greater or equal" through the keyboard; the inequalities in the third line say that the values we are looking for x and y are upper bounded by the given values for A and B, respectively. The third argument gives the set of variables for which we want to know the corresponding values which maximize the function under optimisation.

In response, the software system writes:

```
{ 85, { x -> 80, y -> 5 } }
```

saying that the maximum of the given function is 85, attained for $x = 80$ and $y = 5$. □

Two lots with two contaminants. Suppose as above that we have a pair of cereal lots, but now let us consider a situation in which those lots contain two varieties of contaminants —say they are ammonium and colorant. Here we also want to determine how much of each lot must be used to form the largest amount of mixture, in which each contaminant concentration does not exceed a corresponding given limit.

Again, let A and B denote the corresponding weights, in metric tons, of the given lots. Suppose a_1 and b_1 are the ammonium concentrations in these lots, respectively, and suppose a_2 and b_2 are the corresponding colorant concentrations (Jelinek, 1987). In addition, let k_1 and k_2 be the maximum concentrations permissible in the final mixture, respectively for ammonium and colorant.

As in our first example, here we want to optimise (1). Now we have the following constraints, one for each contaminant, and can be obtained separately by exactly the same reasoning that led us to (2):

$$\begin{aligned} (k_1 - a_1)x + (k_1 - b_1)y \geq 0 \\ (k_2 - a_2)x + (k_2 - b_2)y \geq 0 \end{aligned} \tag{4}$$

Some representative situations that can happen are illustrated graphically in Figure 2. In each graph, the shaded region —which in the third case is reduced to the single point P— represents the solution space for the problem with restrictions, and the point P corresponds to the optimal solution.

Example 3 Here we present three comparative numerical examples whose graphical interpretations are in correspondence with the three cases shown in Figure 2. Again, we assume that the weights are in tons and the concentrations in mg/ton. Observe that in the three cases one lot is slightly contaminated with ammonium and the other lot is also slightly contaminated but with colorant.

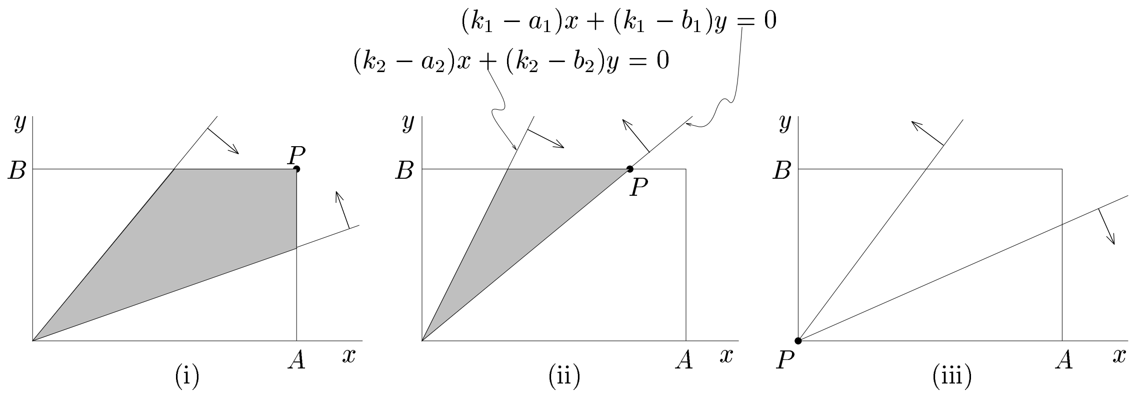

Figure 2: *Two lots with two contaminants*

(i) Let $A = B = 100$, $k_1 = 20$, $a_1 = 35$, $b_1 = 4$, $k_2 = 100$, $a_2 = 45$, and $b_2 = 150$. Proceeding as in Example 2, we type the following input:

```
ConstrainedMax[  x+y,
                 { (20-35)x + (20-4)y >= 0,
                   (100-45)x + (100-150)y >= 0,
                   x <= 100, y <= 100 },
                 { x, y } ]
```

and then we receive the following answer:

```
{200, {x -> 100, y -> 100}}
```

So, the totality of the two given lots can be mixed. We can interpret this by saying that the first lot dilutes the ammonium of the other lot, which at the same time dilutes the colorant of the first one.

(ii) Let the data be as in (i), but with $b_1 = 6$. Now we write:

```
ConstrainedMax[  x+y,
```

```
{ (20-35)x + (20-6)y >= 0,
  (100-45)x + (100-150)y >= 0,
  x <= 100, y <= 100 },
{ x, y } ]
```

and the software system responds with:

```
{580/3, {x -> 280/3, y -> 100}}
```

Hence, in this case the optimal mix weights $580/3 \approx 193.33$ tons, composed of $280/3 \approx 93.33$ tons taken out from the first lot, and the complete second lot.

(iii) Let the data be as in (i), but with $b_1 = 15$ and $a_2 = 70$. To the input:

```
ConstrainedMax[  x+y,
                 { (20-35)x + (20-15)y >= 0,
                   (100-70)x + (100-150)y >= 0,
                   x <= 100, y <= 100        },
                 { x, y } ]
```

we obtain in response:

```
{0, {x -> 0, y -> 0}}
```

This indicates that the only possible "mixture" satisfying 4 is the *null mix*, which weights 0 tons, i.e. no mixture at all can be produced. So, in contrast with case (i) and in some sense contrary to the intuition, neither the first lot can dilute the ammonium of the second lot nor this last lot can dilute the colorant of the first one, simultaneously.

$\square$

Three lots with two contaminants. Here we continue the generalisation process of our optimisation problem, by analysing the case in which we have three cereal lots containing two contaminants.

So, let the three given lots have corresponding weights, in tons, denoted by A, B and C. Each possible mixture will be composed of x, y and z tons, taken out from these lots, respectively. Such a mixture can be geometrically represented by a point (x, y, z) pertaining to the rectangular box shown in Figure 3.

As in the preceeding problem, let k_1 and k_2 be the maximum concentrations of the two contaminants permissible in the final mixture, respectively. Suppose a_1, b_1, and c_1, are the corresponding concentrations of the first contaminant in the given lots, and let a_2, b_2, and c_2, be the concentrations of the other contaminant.

The constraints that the final mixture must satisfy are:

$$\begin{aligned} (k_1 - a_1)x + (k_1 - b_1)y + (k_1 - c_1)z \geq 0 \\ (k_2 - a_2)x + (k_2 - b_2)y + (k_2 - c_2)z \geq 0 \end{aligned} \tag{5}$$

and each one can be deduced in a similar way as the one followed to obtain (2). Under these restrictions, we are looking for the mixture of maximum weight, that is, we want to maximize the function whose value is given by

$$x + y + z \tag{6}$$

In Figure 3 we present a graphical interpretation for a typical case of this problem. Each point of the region S comprised between the two shadowed planes represents a mixture whose contaminant concentrations are under the corresponding given limits.

The optimal mixture —the one with the biggest weight— is represented by point P.

Next we give a numerical example matching this situation.

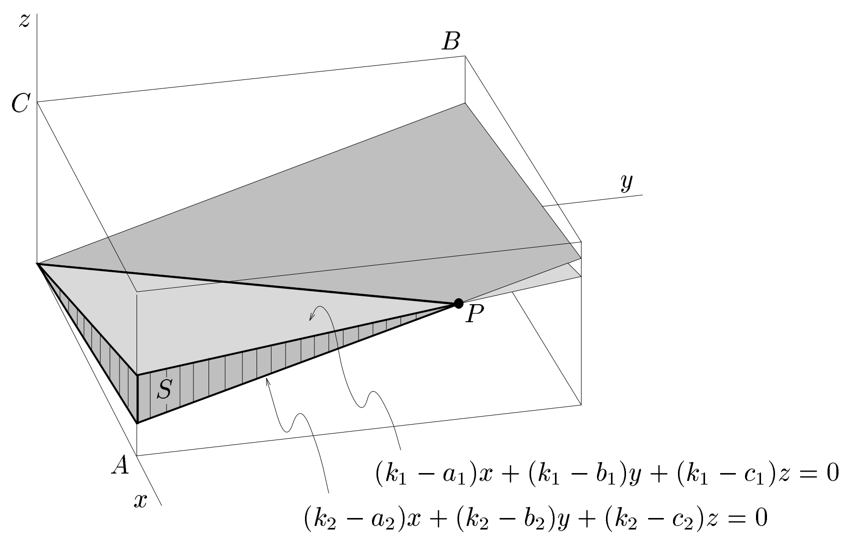

Figure 3: *Three mixtures with two contaminants*

Example 4 Let $A = B = C = 10$ tons and suppose the concentrations, in mg/ton, are: $k_1 = 120$, $a_1 = 140$, $b_1 = 190$, $c_1 = 20$, $k_2 = 80$, $a_2 = 30$, $b_2 = 50$, and $c_2 = 180$.

To solve the problem by means of the beforehand mentioned software system, we type the following lines:

```
ConstrainedMax[  x+y+z,
                 { (120-140)x + (120-190)y + (120-20)z >= 0,
                   (80-30)x + (80-50)y + (80-180)z >= 0,
                   x <= 10, y <= 10, z <= 10 },
                 { x, y, z } ]
```

and in response we obtain:

```
{99/4, {x -> 10, y -> 15/2, z -> 29/4}}
```

Thus, the largest amount of mixture we can obtain weighs 24.75 tons, and is composed of 10, 7.5, and 7.25 tons, respectively taken out from the given lots. □

Generalisation to n lots with m contaminants Suppose we have given n cereal lots with corresponding weights $A_1, \ldots, A_n$, containing m contaminants. Let $a_{i,j}$ represents the concentration of the i-th contaminant in the j-th lot, for $i = 1, \ldots, m$ and $j = 1, \ldots, n$.

As in the preceeding problems, we want to know how much of each lot must be used to form the largest amount of mixture, but in which the concentrations of the contaminants does not exceed the given values $k_1, \ldots, k_m$, respectively.

Let $x_1, x_2, \ldots, x_n$ denote the corresponding quantities of cereal, taken out from the lots to form a mix. We want to optimise the value of the sum

$$x_1 + x_2 + \cdots + x_n \tag{7}$$

subject to the following m restrictions, which can be obtained in essentially the same way used in the particular cases treated before:

$$\begin{array}{rcl} (k_1 - a_{1,1})x_1 + \ (k_1 - a_{1,2})x_2 + \cdots + \ (k_1 - a_{1,n})x_n & \geq & 0 \\ & \vdots & \\ (k_m - a_{m,1})x_1 + (k_m - a_{m,2})x_2 + \cdots + (k_m - a_{m,n})x_n & \geq & 0 \end{array} \tag{8}$$

Observe now that if the number of cereal lots is greater than three, we can not give graphical interpretations as in the particular cases treated before. But algebraically the problem is treated in the same way, as is illustrated in the next example.

Example 5 Let $A_1 = A_2 = A_3 = A_4 = A_5 = 1000$, and the different concentrations be the following:

$$k_1 = 400, \quad a_{1,1} = 300, \quad a_{1,2} = 500, \quad a_{1,3} = 350, \quad a_{1,4} = 600, \quad a_{1,5} = 350,$$

$$k_2 = 200, \quad a_{2,1} = 300, \quad a_{2,2} = 130, \quad a_{2,3} = 180, \quad a_{2,4} = 180, \quad a_{2,5} = 250,$$

$$k_3 = 700, \quad a_{3,1} = 650, \quad a_{3,2} = 750, \quad a_{3,3} = 900, \quad a_{3,4} = 750, \quad a_{3,5} = 550.$$

Proceeding again as in the above examples, we write the following input:

```
ConstrainedMax[ x1+x2+x3+x4+x5,
  { (400-300)x1+(400-500)x2+(400-350)x3+(400-600)x4+(400-350)x5 >= 0,
    (200-300)x1+(200-130)x2+(200-180)x3+(200-180)x4+(200-250)x5 >= 0,
    (700-650)x1+(700-750)x2+(700-900)x3+(700-750)x4+(700-550)x5 >= 0,
    x1 <= 1000, x2 <= 1000, x3 <= 1000, x4 <= 1000, x5 <= 1000 },
  { x1, x2, x3, x4, x5 } ]
```

and in response we obtain:

```
{109000/37, {x1 -> 12000/37, x2 -> 1000, x3 -> 21000/37,
                x4 -> 2000/37, x5 -> 1000}}
```

Then, the best mix is composed of $x_1 = 12000/37 \approx 324.32$, $x_2 = 1000$, $x_3 = 21000/37 \approx 567.57$, $x_4 = 2000/37 \approx 54.05$, $x_5 = 1000$ tons from the corresponding lots, making a total amount of $109000/37 \approx 2945.95$ tons. □

We have solved the dilution problem of contaminants in feeds. In conclusion, although the problems presented here are of known solution, the mathematical method has not been applied in practice. There are other fields where linear programming is usual, as in nutrition and in engineering, but in the biological science this method is not common. The present study shows its application to obtain the solution of daily problems in Toxicology. Although, at the beginning the methodology was tedious and laborious, now with the development of computation and programming languages, particularly those for doing symbolic algebra, it is quite simple to solve such problems; all is reduced to the algebraic formulation of the problem followed by a simple transcription to the computer.

References

Abell, M. L. and Braselton, J. P., 1992. Mathematica by Example. Academic Press, San Diego, California, U.S.A., 647 p.

Bhat, R. V., 1987. Review of activities in mycotoxin prevention and control: Strategies for improvement based on experience in Asia and East Africa. Item 8. Bangkok, Thailand. *Joint FAO/WHO/UNEP Second International Conference on Mycotoxins*, pp. 1–16.

Coker, R. D., Nagler, M. J., Blunden, G., Sharkey, A. J., Defize, P. R., Derksen, G. B., and Whitaker, T. B., 1995. Design of sampling plans for mycotoxins in foods and feeds. *Mycotoxins and Toxic Plant Components,* edited by H.P. Van Egmond, A. Visconti, A. Boenke, G.J.A. Speijers, volume 3, Wiley-Liss, New York, U.S.A., pp. 257–262.

Dichter, C. R., 1987. Análisis costo-eficacia de los programas de control de las aflatoxinas. Tema 12. Bangkok, Tailandia. *Segunda conferencia internacional conjunta FAO/OMS/PNUMA sobre micotoxinas*, pp. 1–8.

Dickens, J. W. and Whitaker, T. B., 1982. Sampling and sample preparation. In *Environmental carcinogens. Selected methods of analysis*, volume 5, IARC Sci. Publ., Lyon, France., pp. 17–32.

Dickens, J. W. and Whitaker, T. B., 1986. Sampling and sample preparation methods for mycotoxin analysis. *Modern Methods in the analysis and structural elucidation of mycotoxins,* edited by R. Cole, Academic Press, U.S.A., pp. 29–49.

Egmond, H. P. V., 1989. Current situation on regulations for mycotoxins. Overview of tolerances and status of standard methods of sampling and analysis. *Food Additives and Contaminants*, 6(2):139–188.

Gass, S. I., 1964. Linear Programming: Method and Applications. McGraw-Hill, New York, U.S.A., 280 p.

Hadley, G., 1962. Linear Programming. Addison-Wesley, Reading, Massachusetts, U.S.A., 520 p.

Jelinek, C. F., 1987. Distribución de las micotoxinas: Análisis de los datos mundiales sobre productos básicos, incluidos los datos del Programa conjunto internacional FAO/OMS/PNUMA sobre vigilancia de la contaminación de los alimentos. Tema 5. Bangkok, Tailandia. *Segunda conferencia internacional mixta FAO/OMS/PNUMA sobre micotoxinas*, pp. 1–318.

Ralston, A., 1993. Simplex method. *Encyclopedia of Computer Science,* edited by A. Ralston, and Reilly, E. D., Van Nostrand Reinhold, New York, U.S.A., 3rd edition, pp. 1202–1204.

Smythe, W. R. and Johnson, L. A., 1966. Introduction to Linear Programming, with Applications. Prentice-Hall, Englewood Cliffs, N.J., U.S.A., 221 p.

Strang, G., 1986. Introduction to Applied Mathematics. Wellesley-Cambridge, Wellesley, Massachusetts, U.S.A., 758 p.

Wolfram, S., 1988. Mathematica: a system for doing mathematics by computer. Addison-Wesley, Redwood City, California, U.S.A., 749 p.

Zionts, S. and Ramesh, R., 1993. Mathematical programming. *Encyclopedia of Computer Science,* edited by A. Ralston, and Reilly, E. D., Van Nostrand Reinhold, New York, U.S.A., 3rd edition, pp. 820–826.