Conservation principles and action schemes in the synthesis of geometric concepts

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Diagrammatic reasoning

- Reasoning
- Learning
- Perception
- Design and creativity
- Theorem proving
- Ubiquitous in science and engineering

2

Diagrammatic reasoning

- How diagrammatic knowledge is represented
- What kind of inferences are supported by diagrams
- How external representations participate in this process

This is a problem in knowledge representation!

3

Some general questions about diagrams

- What is their expressive power
- Why can they be interpreted so effectively
- What is the relation between logic and diagrammatic reasoning

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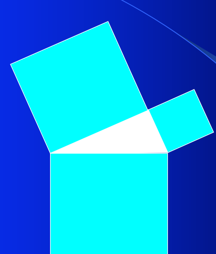
Theorem of Pythagoras



$$h^2 = a^2 + b^2$$

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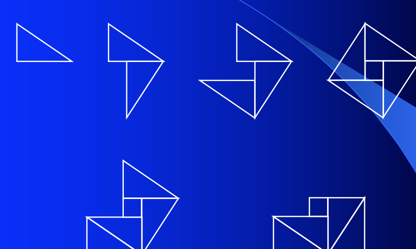
Bronowsky's proof



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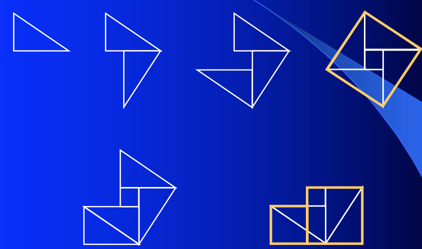
Bronowsky's proof



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What are the mechanisms involved in this kind of reasoning?

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A Challenge for AI

- Gelenter's GTP (late 50's): no account!
- Pineda (1989): The role of reinterpretations
- Barwise and Etchemendy: To illustrate heterogeneous reasoning (1990)
- Wang (1995): The need for generic descriptions
- Lindsay (1998): A demonstrator system
- Jamnik (1999): To illustrate a taxonomy of diagrammatic theorems

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A Challenge for AI

- Pineda (2007):
  - A theory of diagrammatic reasoning
  - A semi-automatic proof of the theorem of Pythagoras
  - A semi-automatic proof of the theorem of the sum of the odds
  - A prototype program

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The theory…

- Action schemes (a synthetic machinery)
- A notion of *re*-interpretation
- A geometric description machinery
- Conservation principles
- The arithmetic interpretation

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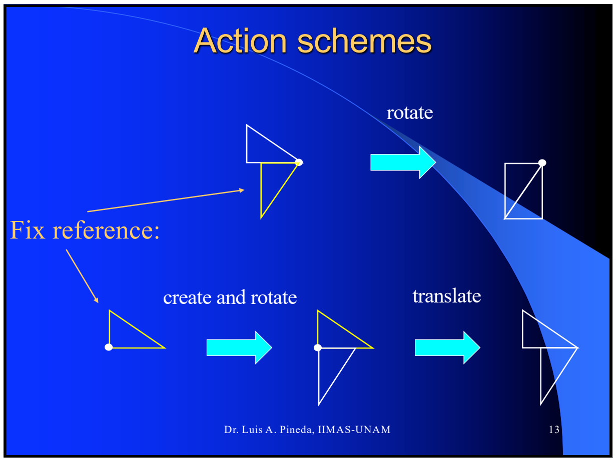

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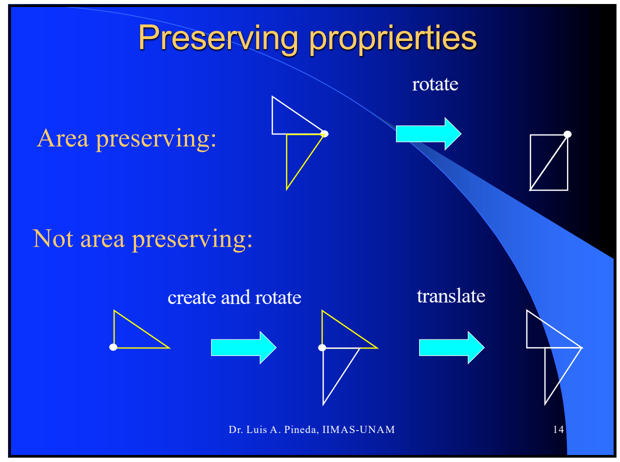

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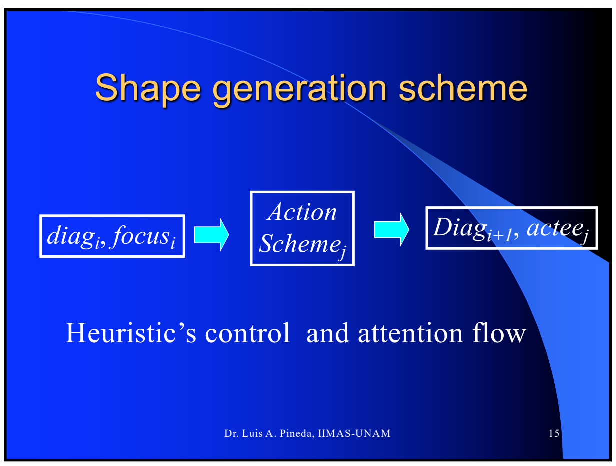

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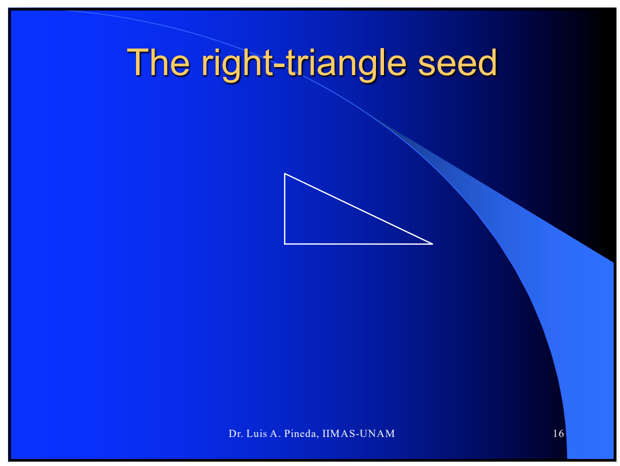

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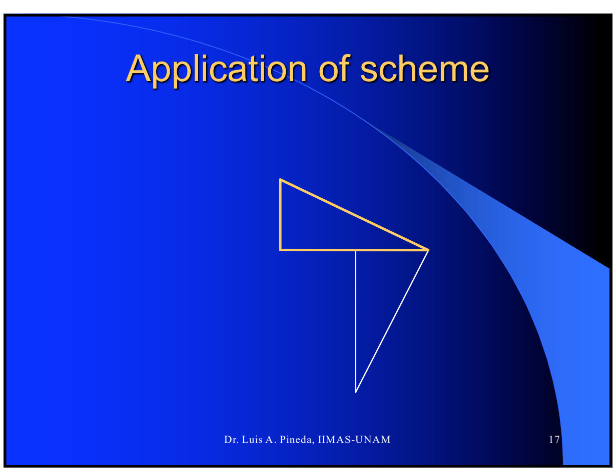

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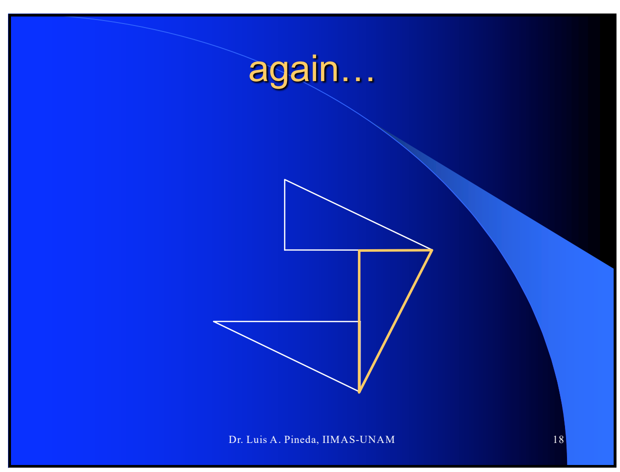

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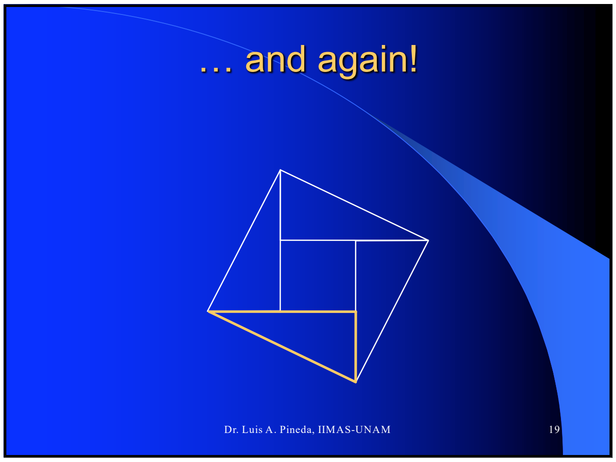

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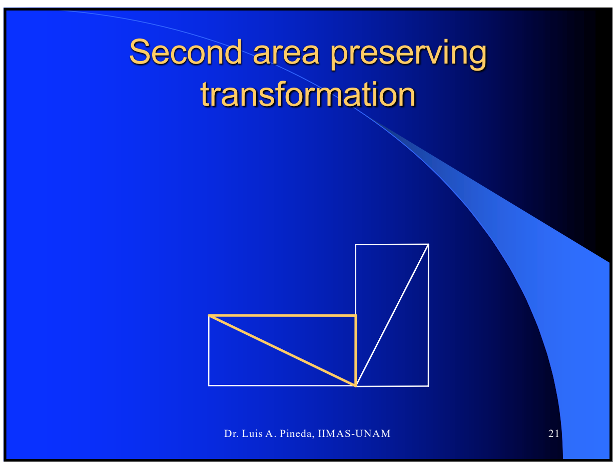

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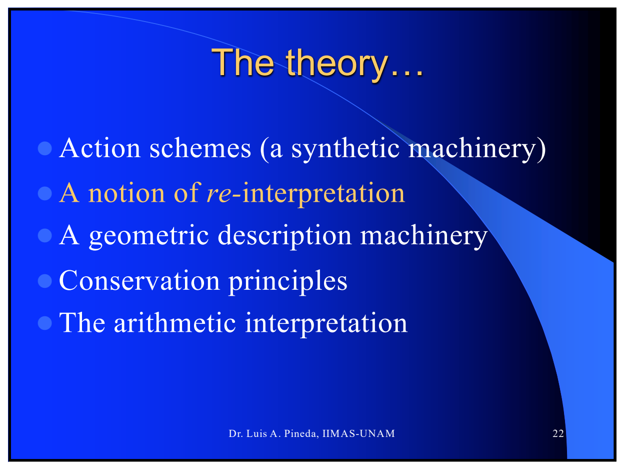

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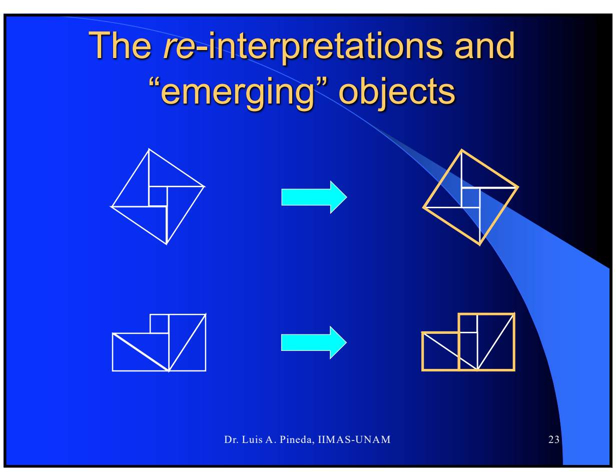

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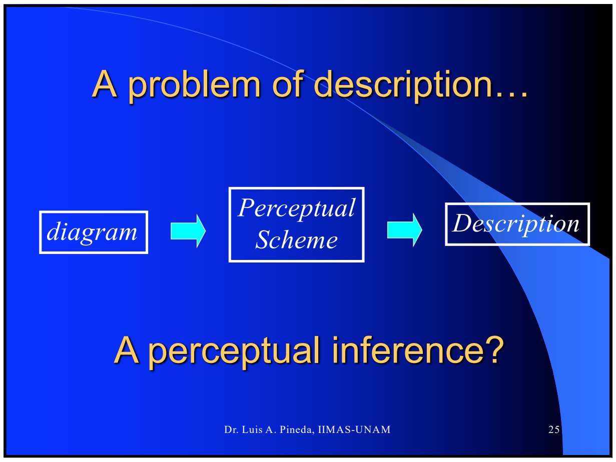

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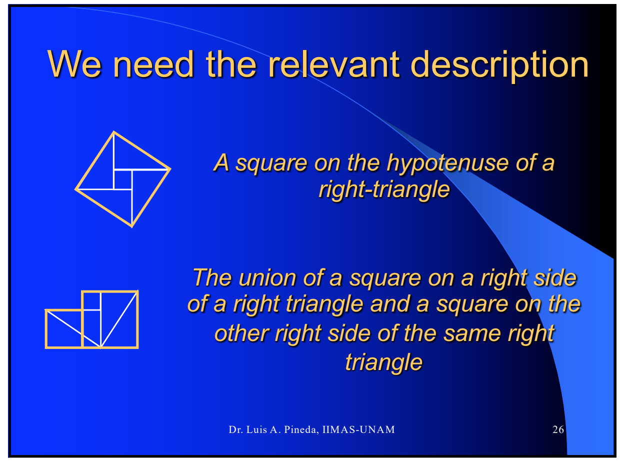

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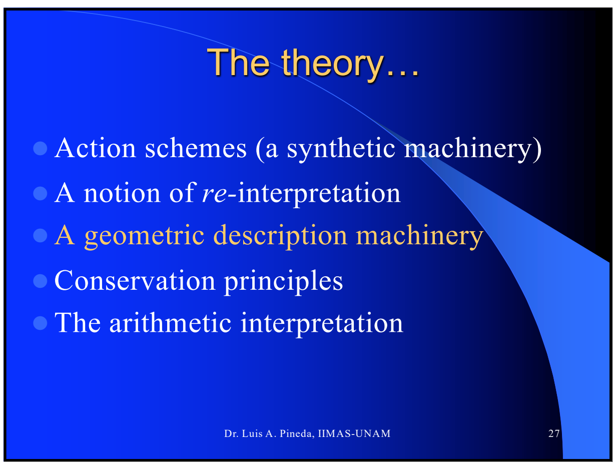

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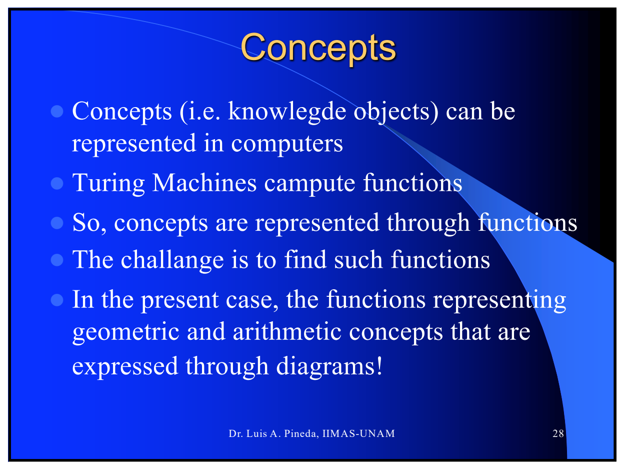

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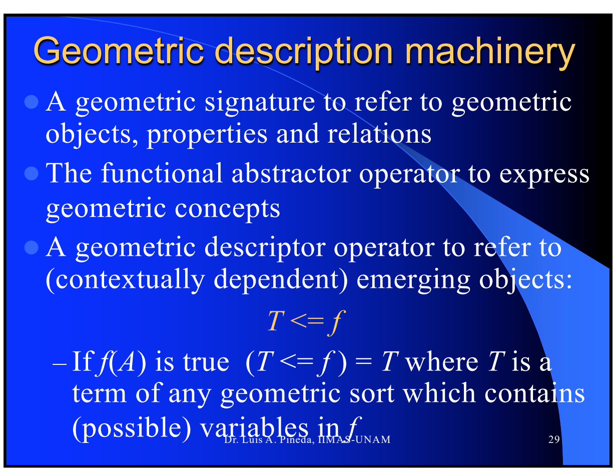

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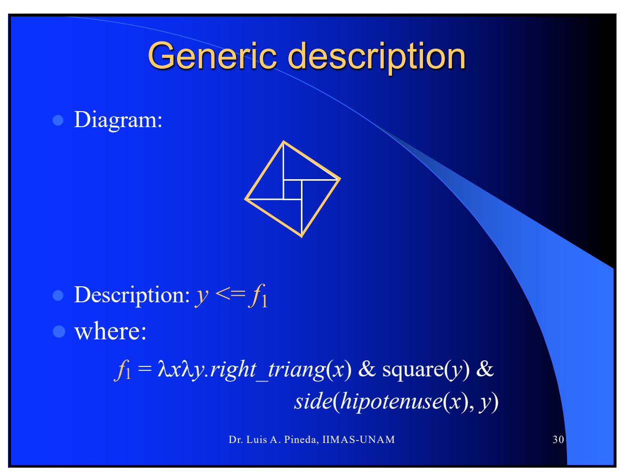

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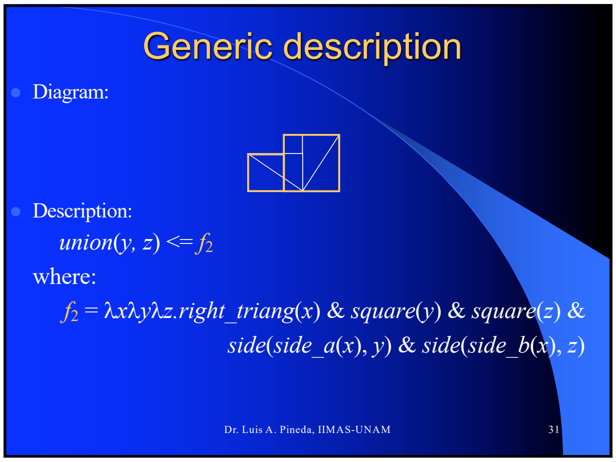

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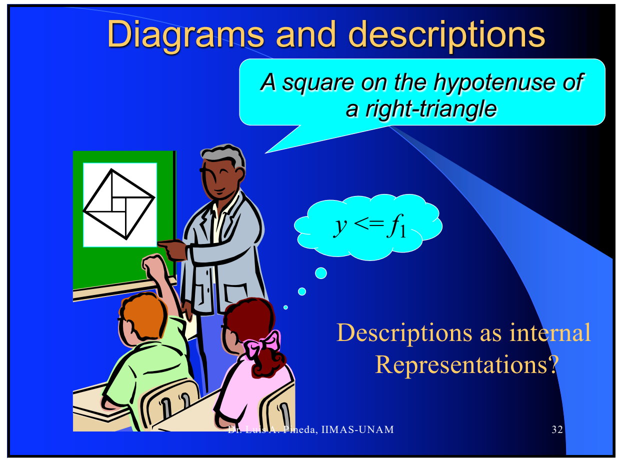

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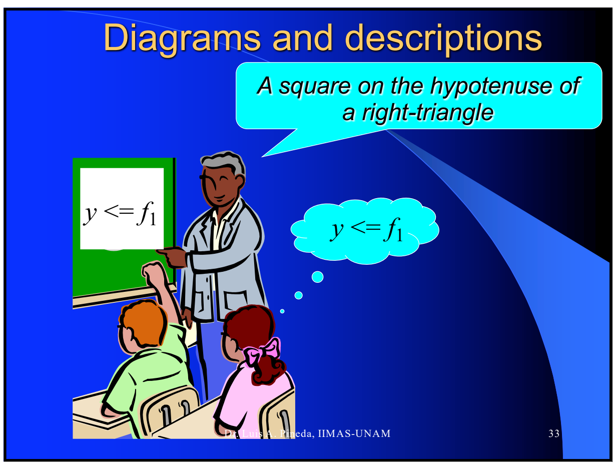

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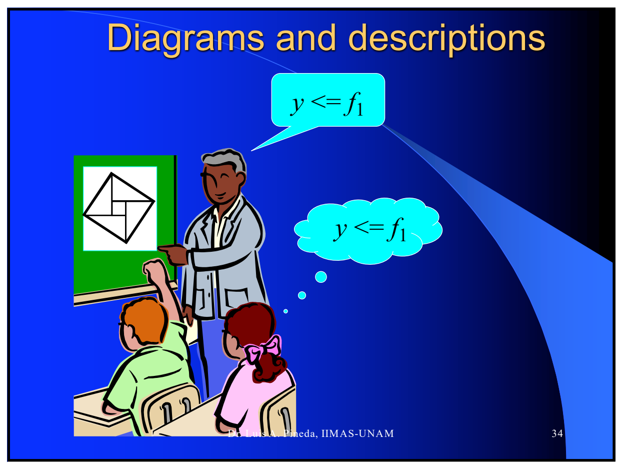

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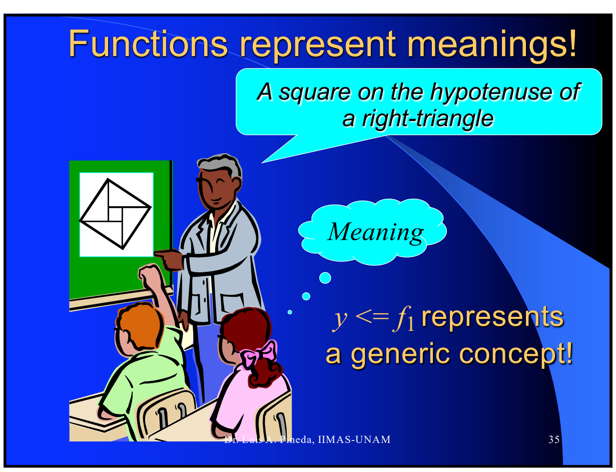

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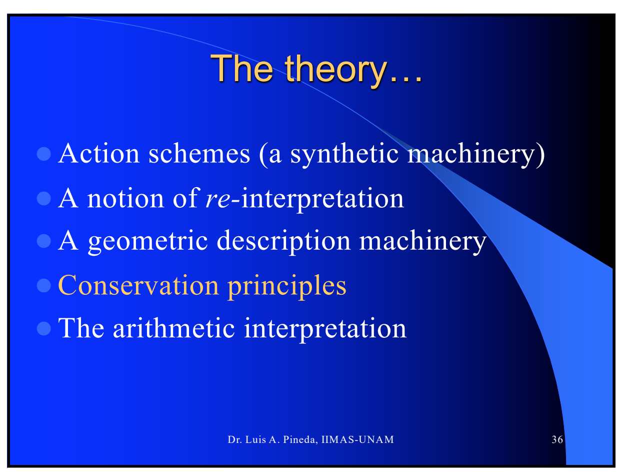

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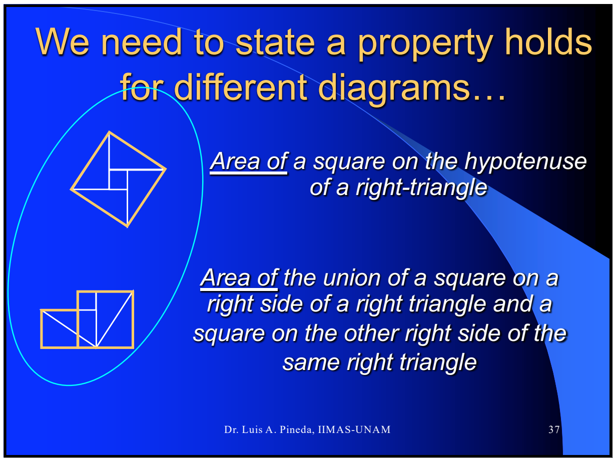

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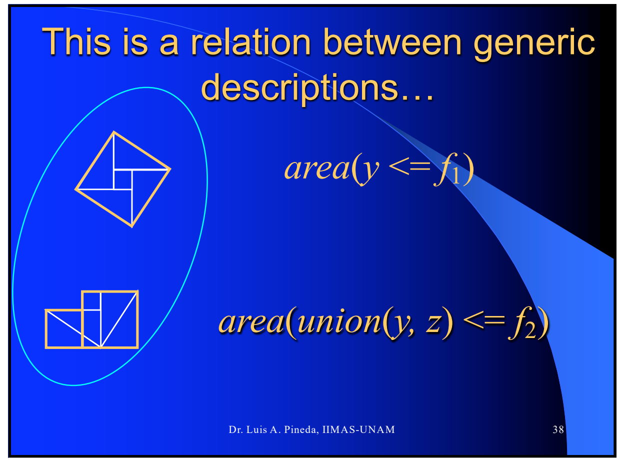

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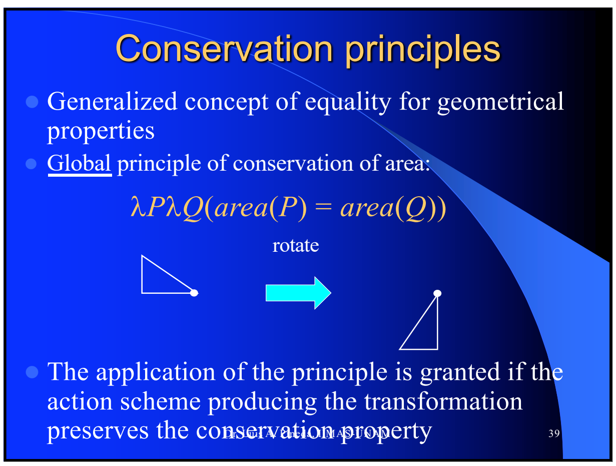

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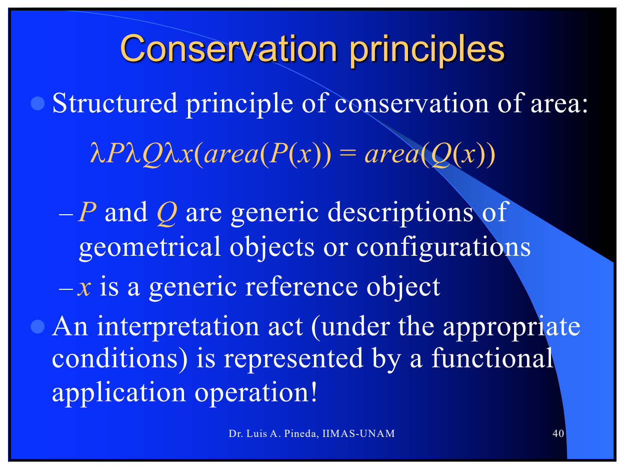

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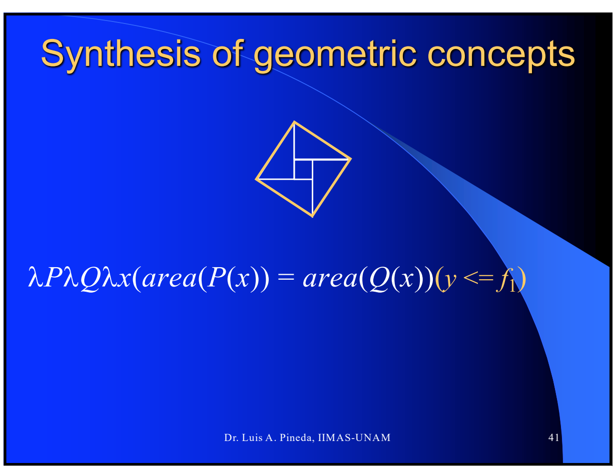

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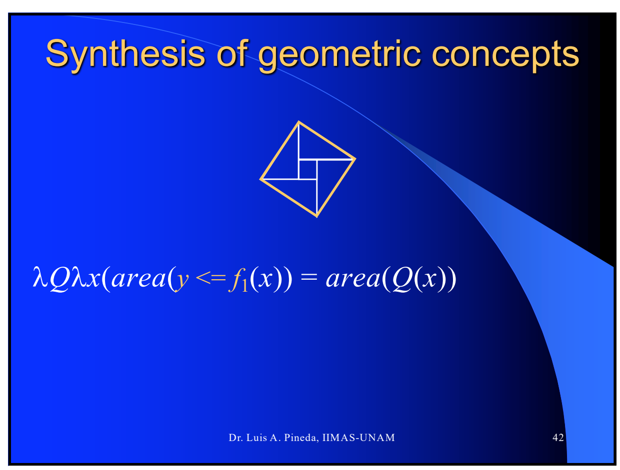

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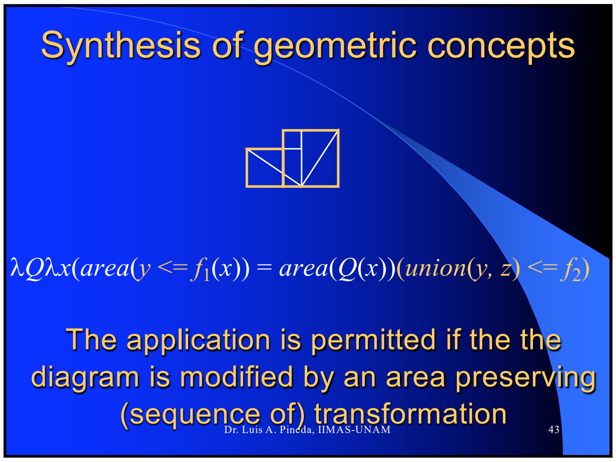

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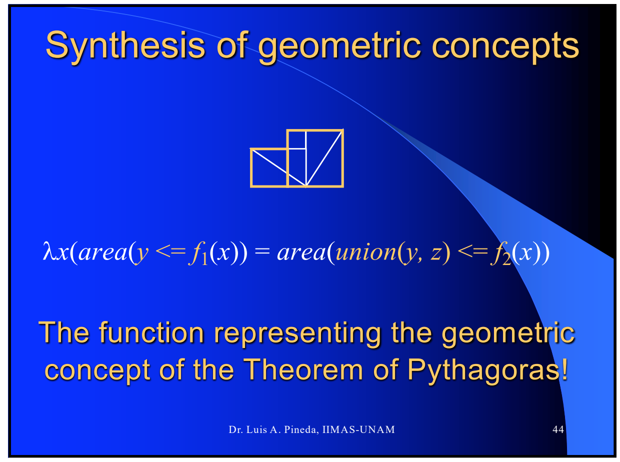

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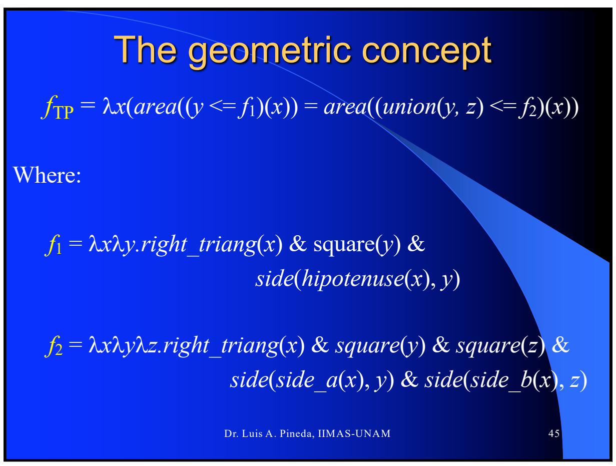

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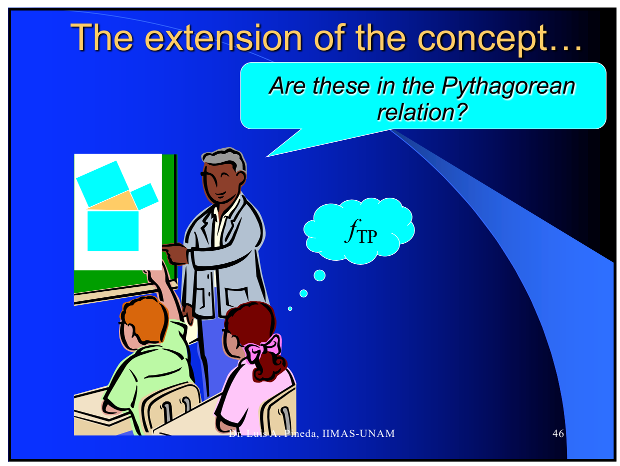

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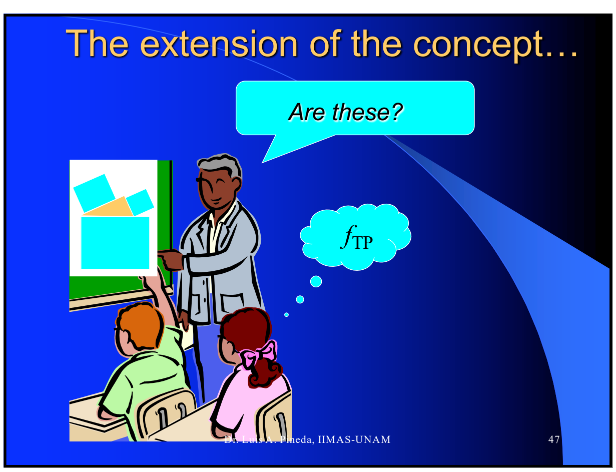

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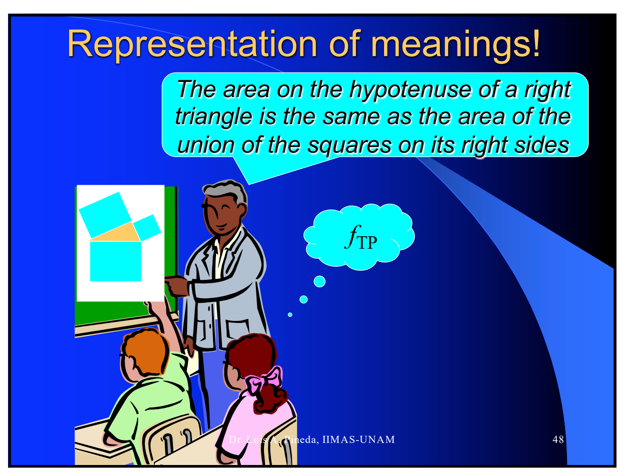

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The theory…

- Action schemes (a synthetic machinery)
- A notion of *re*-interpretation
- A geometric description machinery
- Conservation principles
- The arithmetic interpretation

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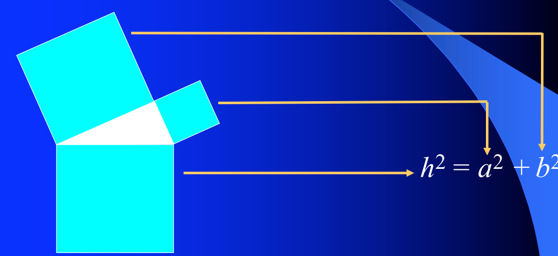
A mapping from the geometry into the arithmetic



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The representation function

$\phi(x \mathrel{<=} f) = \lambda u.u^2$ if the type of x *in* f is *sq*

$\phi(union) = +$

$\phi(g(y_1, y_2) \mathrel{<=} f) = \phi(g)(\phi(y_1 \mathrel{<=} f), \phi(y_2 \mathrel{<=} f))$

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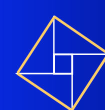
The mapping

Diagram:



The arithmetic concept:

$\phi(y \mathrel{<=} f_1) = \lambda u.u^2$

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The mapping…

The diagram:

The arithmetic concept:

$\phi(union(y, z) \mathrel{<=} f_2) = \lambda v.v^2 + \lambda w.w^2$

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The mapping

- The geometric principle:
  - $\lambda P \lambda Q \lambda x(area(P(x)) = area(Q(x)))$
- The arithmetic principle:
  - $\lambda P \lambda Q(P = Q)$
  - Concept of <u>global</u> aritmetic equality!

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Diagrammatic Derivations

A three-tier tandem process

- The synthesis of geometric form
- The synthesis of the geometric concept
- The synthesis of the arithmetic concept

The seed...

- Diagram:

- Principle of conservation of area:
  – $\lambda P \lambda Q \lambda x(area(P(x)) = area(Q(x))$
- Concept of the global arithmetic equality:
  – $\lambda P \lambda Q(P = Q)$

Synthesis of form

- Diagram:

- Principle of conservation of area:
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- Concept of the global arithmetic equality:
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Synthesis of form

- Diagram:

- Principle of conservation of area:
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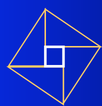

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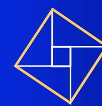
First reinterpretation

- Reinterpretations preserve area:



- Concepts construction:
  - $\lambda P \lambda Q \lambda x(area(P(x)) = area(Q(x))(w <= f_1)$
  - $\lambda P \lambda Q(P = Q)(\lambda u.u^2)$

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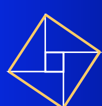
First reinterpretation

- Reinterpretation:



- Concepts construction:
  - $\lambda Q \lambda x(area(w <= f_1(x)) = area(Q(x))$
  - $\lambda Q(\lambda u.u^2 = Q)$

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Synthesis of form

- Diagram:

- Concepts construction:
  - $\lambda Q \lambda x(area(w <= f_1(x)) = area(Q(x))$
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Synthesis of form

- Diagram:

- Concepts construction:
  - $\lambda Q \lambda x(area(w <= f_1(x)) = area(Q(x))$
  - $\lambda Q(\lambda u.u^2 = Q)$

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Second reinterpretation

- Reinterpretation:

- Concepts construction:
  - $\lambda Q \lambda x(area(w <= f_1(x)) = area(Q(x))(union(y, z) <= f_2)$
  - $\lambda Q(\lambda u.u^2 = Q)(+(\lambda v.v^2, \lambda w.w^2))$

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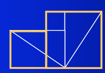
Second reinterpretation

- Reinterpretation:



- Concepts construction:
  - $\lambda x(area(w <= f_1(x)) = area(union(y, z) <= f_2(x))$
  - $\lambda u.u^2 = +(\lambda v.v^2, \lambda w.w^2)$

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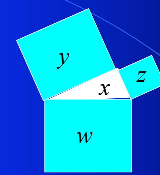
Program transformation rules



$$\lambda x.\lambda w.\lambda y,z.(area((w <= f_1)(x,w)) = area((union(y, z) <= f_2)(x,(y, z)))$$

$$\lambda u,v,w.u^2 = v^2 + w^2$$

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Questions about diagrams

- What is their expressive power
- Why can they be interpreted so effectively
- What is the relation between logic and diagrammatic reasoning

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Questions about diagrams

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Diagrams and abstraction

- A common view is that diagrams are good for expressing concrete information but…
- There is a limitation in the abstractions that can be expressed
- The theory of graphical specificity (Stenning and Oberlander, 1995)

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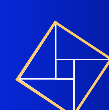
We can have concrete interpretations…

- Diagram:



- Description:

$sq_1 <= right\text{-}triangle(rt_1)$ & $square(sq_1)$ & $side(hipotenuse(rt_1), sq_1)$

… and deal with the ambiguity!

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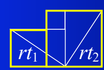
We can limit the expressive power of the representational language…

Diagram:



Descripción:

$union(sq_1, sq_2) \Leftarrow right\_triang(rt_1)\ \&\ right\_triang(rt_2)\ \&$
$square(sq_1)\ \&\ square(sq_2)\ \&$
$side(side\_a(rt_1), sq_1)\ \&$
$side(side\_b(rt_2), sq_2)$

and face the limitations of the medium!

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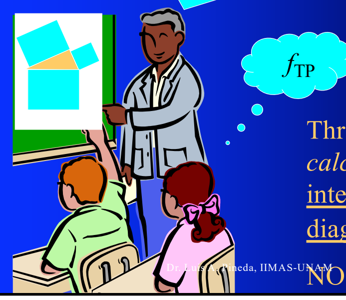
Representation of meanings

The area on the hypotenuse of a right triangle is the same as the area of the union of the squares on its right sides



Through the *lambda calculus* we represent interpretations of diagrams …

NOT diagrams!!!

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Diagrams and abstraction

- The present theory shows that diagrams can be given generic (fully abstract) interpretations!
- A representation is specified through:
  - The external symbols and configurations
  - The interpretation process
  - The language to represent the interpretations does not need to have a limited expressivity (e.g. propositional logic)
- Diagrammatic proofs are genuine proofs!

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Questions about diagrams

- What is their expressive power
- Why can they be interpreted so effectively
- What is the relation between logic and diagrammatic reasoning

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Reasoning with concrete representations

- Vision provides concrete interpretations of shapes directly
- Easy… if the problem has a concrete nature!
- Concrete problems can often be expressed through diagrams
- But, if the problem demands abstraction (e.g. an infinite number of instances) concrete resources (memory and computational time) run out very quickly!

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Abstractions capture change implicitly!

- Two dimensions of change:
  - The parameters of the diagrammatic objects
  - Different diagrammatic configurations that have the same description (i.e. equivalent in relation to the task)

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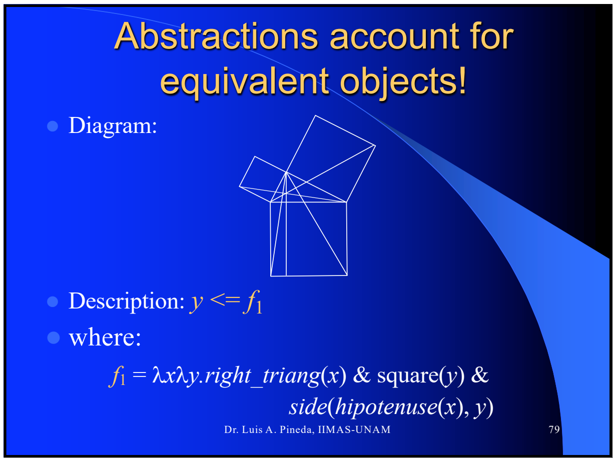

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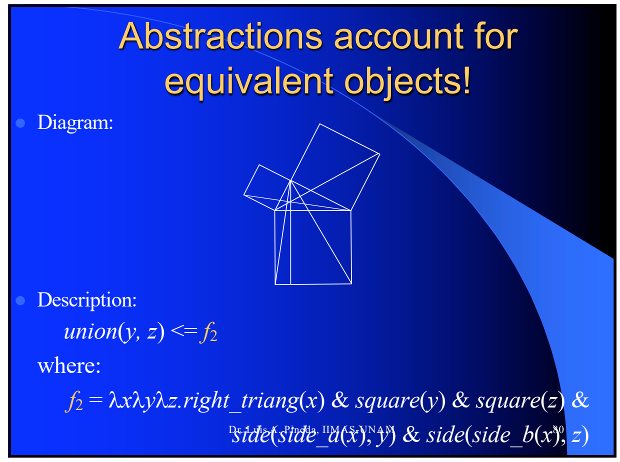

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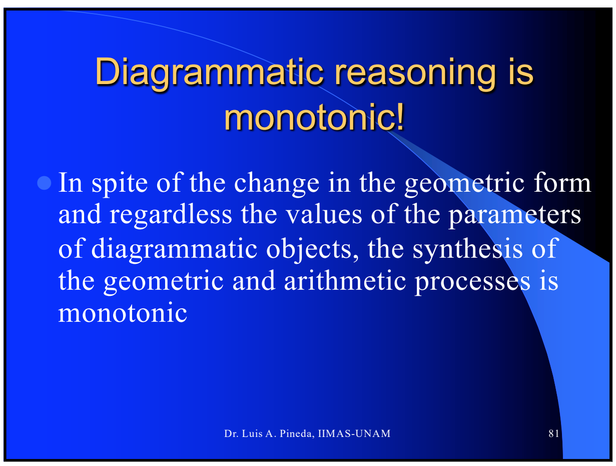

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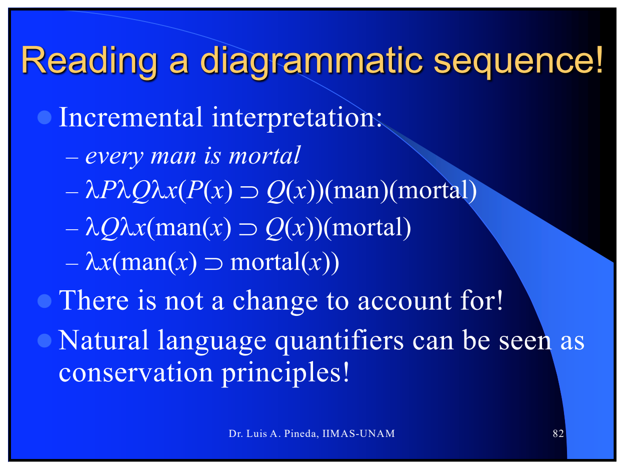

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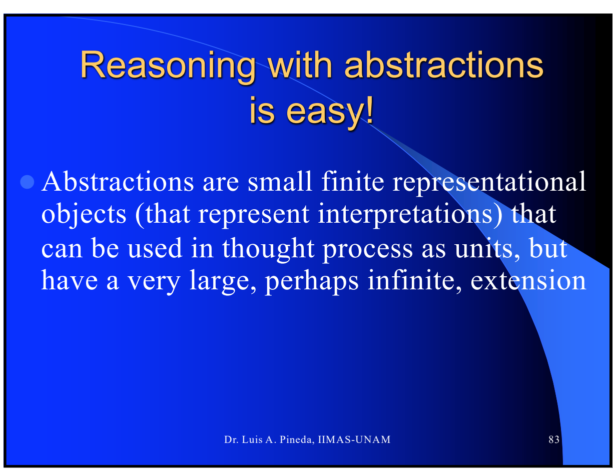

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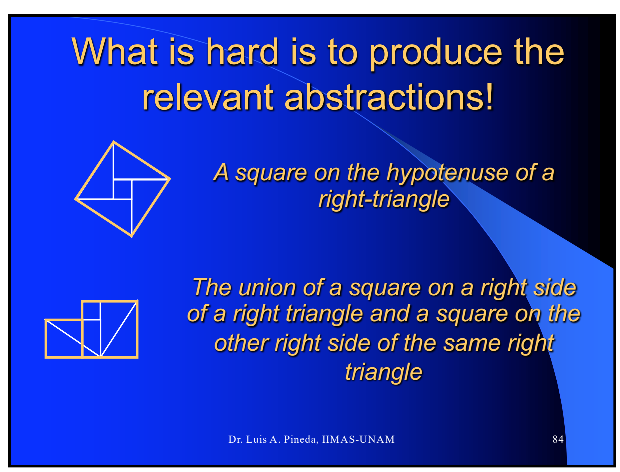

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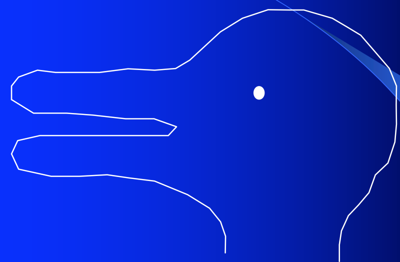
Abstracting on concrete descriptions?



$duck_1 <= duck(duck_1)$?

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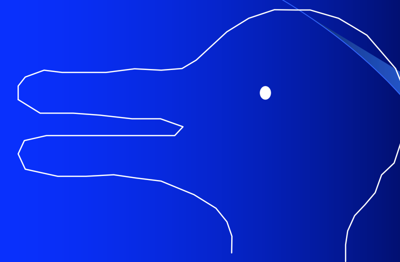
Constructing the abstraction directly!



$x <= \lambda x.duck(x)$

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Generation of abstract descriptions …

- The extensional representation
- Visualisations (i.e. Reinterpretations)
- Domain knowlege (e.g. Geometry)
- Knowledge about the aims of the task (e.g. theorem proving and discovery)

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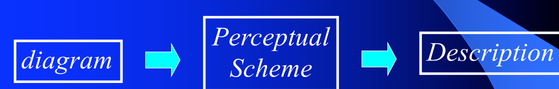
Perceptual inference



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Questions about diagrams

- What is their expressive power
- Why can they be interpreted so effectively
- What is the relation between logic and diagrammatic reasoning

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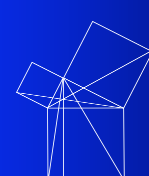
The axiomatic method



Proposition 47, Euclid's Elements

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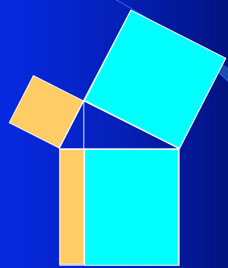
The axiomatic method



A simpler problem!

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Reinterpretations

- Enrich the problem-solving space
- Interesting emerging objects belong to the enriched space
- The recognition of emerging objects depends on the interpretation process, but also on the nature of the external representation!
- The process is genuinely synthetic and synthesized objects cannot be found through analysis!

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The paper:

Luis A. Pineda, Conservation principles and action schemes in the synthesis of geometric concepts, *Artificial Intelligence* 171 (March, 2007) 197-238.

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Thanks very much!

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